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Pre-Service Elementary Mathematics Teachers' Informal Strategies for Multiplication and Division of Fractions

İlköğretim Matematik Öğretmen AdaylarınınKesirlerde Çarpma ve Bölmede Kullandıkları İnformal Stratejiler

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Abstract

The aim of the study was to identify pre-service elementary mathematics teacher's informal strategies for multiplication and division of fractions. The design of the study is a descriptive survey focusing on pre-service elementary mathematics teachers' strategies. The participants were 173 pre-service elementary mathematics teachers including 45 freshmen, 64 sophomore, and 64 junior students studying in an elementary mathematics education program at a public university in Ankara. In this study, pre-service teachers were asked to solve two computations of fractions questions, $\frac{2}{5} \times 2\frac{1}{2}$ and $\frac{1}{2} \div \frac{1}{8}$, without using the rule-based strategy (e.g. inverting the divisor and multiply). The results of the study revealed that pre-service elementary mathematics teachers used four different strategies (using area model, using set model, repeated addition, and distributive property) for multiplication of fractions; five different strategies (using area model, using set model, common denominator, repeated subtraction, using decimal strategy) for division of fractions. It was also found that the percentages of using these strategies were quite low.

Keywords: Informal strategy, multiplication of fraction, division of fraction, pre-service teachers.

Öz

Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının kesirlerde çarpma ve bölme işlemlerinde kullandıkları informal stratejileri belirlemektir. Çalışma öğretmen adaylarının stratejilerini belirlemeye yönelik olduğundan betimsel tarama desenindedir. Katılımcılar, Ankara'daki bir devlet üniversitesinin ilköğretim matematik öğretmenliği programında öğrenim gören 45'i birinci sınıf, 64'ü ikinci sınıf, 64'ü üçüncü sınıf olmak üzere toplam 173 öğretmen adayından oluşmaktadır. Bu çalışmada öğretmen adaylarından, standart algoritmalar kullanılmadan (örneğin ters çevirip, çarpma algoritması), kesirlerde çarpma ve bölme ile ilgili şu soruları yanıtlanmaları istenmiştir: $\frac{2}{5} \times 2\frac{1}{2}$ ve $\frac{1}{2} \div \frac{1}{8}$.Çalışmanın bulguları, öğretmen adaylarının kesirlerde çarpma işlemi için dört farklı strateji (alan modeli kullanımı, küme modeli kullanımı, tekrarlı toplama ve dağılma özelliği); kesirlerde bölme işlemi için ise beş farklı strateji (alan modeli, küme modeli kullanımı, ortak payda, tekrarlı çıkarma, ondalık gösterim kullanımı) kullandıklarını ortaya çıkarmıştır. Genel olarak bu stratejilerin kullanımının düşük olduğu da belirlenmiştir.

Anahtar Kelimeler: İnformal stratejiler, kesirlerde çarpma, kesirlerde bölme, öğretmen adayları.

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1. Introduction

Fractions and computations with fractions are considered one of the most complex subjects for the students in early years of their school experiences, even in their higher grades (Ball, 1990; Brown & Quinn, 2007; Ma, 1999), since they are complex structures being composed of many relations (Thompson, 1993). Those relations can be constructed with conceptual understanding rather than procedural skills. However, having lack of conceptual understanding usually makes students, and even for teachers, use standard algorithms like inverting the divisor and multiplying fractions or other rules of computation, which are generally expected to be forgotten. In order to develop conceptual understanding, students should be given sufficient number of opportunities to produce their own strategies rather than rules or procedures taught by teachers (Van de Walle, Karp, & Williams, 2007). Those personal or flexible strategies are called as *invented strategies* referring to "any strategy other than the traditional algorithm" (p.218) (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998). In fact, those strategies developed by students, which can be called as *informal strategies*, would then contribute to developing standard algorithms (Huinker, 1998). For division of fractions, for example, students' knowledge should not be limited to the invert-and-multiply algorithm, which is considered the least understood procedure to be taught to the students (NCTM, 2000). Rather, they need to learn beyond algorithmic rules for gaining conceptual understanding, which is considered difficult for both students and teachers. Son and Crespo (2009) identified alternative strategies as *informal strategies* for dividing fractions by analyzing different curriculum, textbooks, materials, and the related literature. They listed them as "common denominator strategy, repeated subtraction strategy, using decimals, using a unit rate, applying distributive law, dividing numerators, and denominators strategy" (p.238).

Division of fractions has multiple interpretations, such as division as measurement, partition, the inverse of a Cartesian product, which are the extensions of whole number interpretations (Van de Walle, Karp, & Williams, 2007). Sinicrope, Mick, and Kolb (2002), however, stated that those interpretations are not enough to explain division of fractions and represented two additional extensions; namely, the determination of a unit rate and division as the inverse of multiplication. Depending on those diverse interpretations, different computational strategies can emerge. The idea is that if teachers could help extend students' interpretations of division by fraction, they might increase their computational flexibility (Son &Crespo, 2009). Ma (1999) argued that pre-service teachers' low performances in computation with fractions, specifically division with fractions, stem from their lack of knowledge about the different meaning of division depending on the given context. Parallel to the division, multiplication of fractions has different meanings reported in the literature. Azim (1995), in her study, represented four models for multiplication of whole numbers: (1) repeated addition, (2) multiplicative compare, (3) area concepts, and (4) Cartesian product. However, it is noted that when these models are extended to fractions, a re-conceptualization is needed for the fractional quantities (Azim, 1995). When two fractions (e.g. $\frac{3}{4} \times \frac{1}{7}$) are multiplied, for instance, it cannot represent repeated addition, since it shows the part of part meaning of multiplication of fractions (Mack, 2000).

There are many studies investigating students and/or pre-service teachers' competency in computation with fractions (Huang, Liu, & Lin, 2009). Izsak (2008) noted that fraction division and decimal multiplication have been studied more while comparing with the studies related to fraction multiplication. Most researches focus on the error analysis of those computations (Brown & Quinn, 2007; Işık & Kar, 2012; Işıksal & Çakıroğlu, 2008; Isiksal & Cakiroglu, 2011). In the following, some studies are presented including pre-service teachers focusing on fractions.

Huang, Liu, and Lin (2009) investigated Taiwanese pre-service teachers' mathematical knowledge in fractions, including their understanding and computational abilities. Pre-service teachers were first asked to "solve the problem $\frac{3}{4} \times \frac{2}{3} =$? and then they were required to give an illustration or representation of the given problem to analyze their conceptions. The findings showed that pre-service teachers performed better fraction knowledge on procedure than on conception. In other words, they demonstrated lack of conceptual knowledge in fractions. Işıksal and Çakıroğlu (2008, 2011) investigated pre-service teachers' knowledge of students' difficulties in division and multiplication of fractions and their preferred strategies to eliminate those difficulties. The strategies were grouped under different headings based on teaching methods and/or formal knowledge of fractions. In other studies in the literature, pre-service teachers were asked to generate word problems corresponding to the fractions written in symbolic form. In his study, Işık (2011) conducted a conceptual analysis of multiplication and division problems in fractions posed by pre-service elementary mathematics teachers. The analysis showed that pre-service teachers' performances were poor in multiplication of mixed fractions and division of two fractions. With a similar aim, Unlu and Ertekin (2012) investigated whether pre-service elementary mathematics teachers could write a word problem for modeling different computations including $\frac{1}{2} \div \frac{1}{3}$, $\frac{1}{2} \div \frac{5}{3}$, $\frac{5}{3} \div \frac{1}{2}$ and $\frac{5}{3} \div \frac{3}{2}$. The overall performances of the participants showed that pre-service teachers did not have adequate knowledge about division of fractions. Toluk-Uçar (2009) conducted an experimental study examining the effect of problem posing on pre-service teachers' understanding of fraction concept including multiplication and division of fractions. The pre-service teachers were asked to calculate the computations such as $\frac{2}{3} \div \frac{1}{2}$, $\frac{3}{4} \times \frac{1}{3}$; write a word problem representing the computations, and show suitable representations for the given statements. The results showed that pre-service teachers in the experimental group in which problem posing was used as a teaching strategy demonstrated better performance. In another study, Gökkurt, Soylu, and Demir (2015) investigated secondary mathematics teachers' opinions on teaching fractions. The findings demonstrated that teachers participated in the study had lack of knowledge in models and using them while teaching fractions. These findings reported in the literature generally illustrate that pre-service teachers' performances were poor in posing problems and using models or representations including multiplication and division of fractions.

Teachers are expected to know different and beneficial ways of using algorithms in order to make students aware of which methods are reasonable to use in different situations (Campbell, Rowan & Suarez, 1998). This expectation is meaningful since computations are considered an important part of mathematics, and they are valuable for student learning. In the related literature, it is argued that pre-service teachers' performances in computation with fractions has a reflecting role on their future students' learning in the same subject (Castro-Rodríguez, Pitta-Pantazi, Rico, & Gómez, 2016; Işık, 2011; Işıksal&Çakıroğlu, 2008; Wahyu, Amin,&Lukito, 2017). In other words, when pre-service teachers acquire conceptual knowledge of any mathematical subject, they most probably reflect their knowledge to their students on the same manner. From this perspective, pre-service teachers' capabilities in using strategies while solving multiplication and division with fractions are considered important in terms of constructing a conceptual understanding for students. Thus, the main purpose of the study was to identify pre-service teacher's informal strategies for multiplication and division with fractions. Those strategies used by pre-service elementary mathematics teachers were also considered according to their grade level. The research question was formulated as in the following:

What are the pre-service elementary mathematics teachers' informal strategies for multiplication and division with fractions?

2. Method

2.1. Design of the Study

In this study, pre-service teachers were asked to solve two computations of fractions questions without using the rule-based strategy (e.g. inverting the divisor and multiply) in order to identify their own personal strategies. The design of the study fits into descriptive survey which focuses on individuals' abilities, preferences, and/or behaviors (Fraenkel&Wallen, 2009). The focus of this study was to identify participants' informal strategies based on the stated informal strategies in the literature.

2.2 Participants

The participants of the study were 173 pre-service elementary mathematics teachers including 45 freshmen, 64 sophomore, and 64 junior students studying in a public university in Ankara. They were selected by using convenience sampling during the spring semester of 2016-2017 academic years. At the time of the data collection procedure, junior students, who were in their sixth semester, had been taking teaching method courses for two semesters as a requirement in the undergraduate program, while freshmen and sophomore students who were in their second and fourth semester, had not taken mathematics teaching methods courses yet. The senior students were not included to the study since they spent most of their time in doing their teaching practices at practicum schools when the data were collected.

2.3. Data Collection Tool and Procedure

The two questions including "$\mathbf{\frac{2}{5} \times 2\frac{1}{2}}$" and "$\mathbf{\frac{1}{2} \div \frac{1}{8}}$", were used as data collection tool in this study. The pre-service teachers were asked to solve these two questions without using standard rule or algorithms, such as multiply numerators and denominators for multiplication of fractions or invert and multiply the fractions for division of fractions. Rather, they were required to do computations by using their own strategies or alternative ways to reach the answers. The data were collected by the first researcher who was also the instructor of the teaching method courses.

The researcher informed the pre-service teachers about the purpose of the study and gave them 20 minutes to do the computations.

2.4. Data Analysis

Data analysis processes started with coding correct and incorrect/no responses by giving 1 and 0, respectively. Then, from among the correct responses, strategies were identified for each question. The identified strategies were named based on the related the literature (Son & Crespo, 2009; Van de Walle et al., 2007). Identified strategies were coded together with both researchers. Thus, they reached a full agreement for the name of strategies. For descriptive statistics, these strategies were coded, and the data were analyzed by using Statistical Package for the Social Sciences (SPSS20.0). Frequencies and percentages of correct and incorrect responses and used strategies based on grade level were calculated and presented in the following part.

3. Results

The aim of this study was to determine informal strategies of pre-service elementary mathematics teachers used in multiplication and division of fractions. First, frequencies and percentages of correct and incorrect/no responses were calculated. As can seen in Table 1, the performances of pre-service teachers were higher in division of fractions than in multiplication of fractions. 72,3% and 50,3% of pre-service teachers could successfully divide and multiply the fractions, respectively. They generally left the questions of multiplication blank and written the explanation "*I cannot answer this without using the standard algorithm*" on their answer sheets.

Table 1
Performances of pre-service teachers in division and multiplication of fractions

	Multiplication of fraction	Division of fraction
Correct answers (%)	87 (% 50,3)	125 (%72,3)
Incorrect or no answers (%)	86 (% 49,7)	48 (% 27,6)
Total	173	173

An example of *incorrect or no answers* for multiplication of fraction taken by a pre-service teacher in her second grade is given in *Figure 1.* As it is also clear in the figure, the pre-service teacher did not consider the size of the whole and part of the same whole. Moreover, she had difficulty in creating a representation for multiplication of fractions.

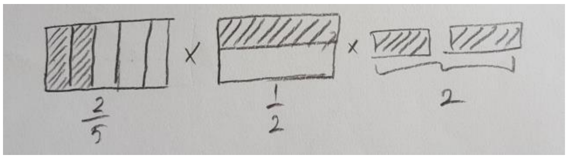


Figure 1. *An example of incorrect response for multiplication of fraction*

As their strategies were examined, it was observed that pre-service teachers used four different strategies for multiplication of fractions and five different strategies for division of fractions except the use of *standard algorithm*. These strategies, their frequencies (percentages), and the explanations for these strategies are exhibited in *Table 2.*

Table 2

The strategies used by pre-service teachers in multiplication and division of fraction

		Strategies	f (%)	Explanation
Multiplication of Fractions	Informal strategies	Using Model a. Area Model	18 (% 10,4)	First, the bigger fraction is illustrated using area model. Then, the smaller fraction is shaded in the illustrated shape.
		Using Model b. Set Model	2 (% 1,2)	The whole is considered as a set of objects. If the whole consists of 10 counters, then 2 represents 25 counters. Then, 3 different sets are formed. Two sets consist of 10 counters and the remaining one set consists of 5 counters. These 3 sets were partitioned into fifths. Two of these parts are shaded.
		Repeated Addition	31 (% 17,9)	The question is solved by using the meaning of repeated addition method of multiplication of whole numbers. The multiplication operation 2 denotes the addition of the fraction twice and adding one half of the product together.
		Distributive Property	17 (% 9,8)	The problem is solved by expressing mixed fraction with whole number and proper fraction and using distributive property of multiplication over addition.
	Standard Algorithm		13 (% 7,5)	The question is solved by multiplying the nominators and writing the result as nominator and multiplying the denominators and showing the result in denominator.
	Incorrect or no Response		92 (% 53,2)	Inappropriate response, incorrect representation or leaving the question blank.
	Total		173 (% 100)	
Division of Fractions	Informal strategies	Using Model a.Area Model	76 (% 43,9)	Firstly, bigger fraction (1/2) is shown. The amount of small fraction (1/8) is looked for within the shape or 8 times ½ is shaded on the model.
		Using Model b. Set Model	6 (% 3,5)	Whole is considered as a set of objects. If a whole equals to 16, one half equals to 8. One eight of the whole represents 2. Then, it is thought that how many groups of 2 can be formed in 8.
		Common Denominator Strategy	11 (% 6,4)	1/2 is rewritten as 4/8 and it is considered as 4 times 1/8, then the number of 1/8 can be found within 4/8.
		Repeated Subtraction	1 (% 0,6)	The solution is obtained using the meaning of division as repeated subtraction in whole numbers.
		Using Decimal Strategy	6 (%3,5)	The solution is obtained by converting the fractions ½ and 1/8 to decimals 0,5 and 0,125; then the nominator is divided by the denominator.
	Standard Algorithm		25 (% 14,5)	The solution is obtained by inverting the second fraction and multiplies it with the first fraction.
	Incorrect or no Response		48 (%27,6)	Inappropriate response, incorrect representation or leaving the question blank.
	Total		173 (%100)	

When the strategies are examined, the most commonly used strategy has been found to be the repeated addition for multiplication of fractions after incorrect/no responses.18% of the pre-service teachers used the meaning of repeated addition in multiplication of whole numbers. Those who used repeated addition strategy solved the question $\mathbf{\frac{2}{5} \times 2\frac{1}{2}}$ by adding the fraction of $\frac{2}{5}$ twice and adding the result with one half of $\frac{2}{5}$. To state the solution mathematically:$\frac{2}{5}+\frac{2}{5}+\frac{1}{5}$. specifically, the pre-service teachers who used this strategy generally wrote statements like “add the 2/5 twice” on their answer sheets.

Using model (area model) is another strategy used more than the repeated addition in multiplication of fractions. For solving the question, 10,4% of the pre-service teachers used this strategy by first considering the bigger fraction, then shading the portion of smaller fraction within the shape of bigger fraction. First of all, they indicated the mixed fraction $2\frac{1}{2}$, by shading two whole areas and a half of the whole, and they got the solution by shading the two fifth of each whole. The difference between this strategy and the repeated addition of fractions is that the big fraction is shaded as the amount of proper fraction in the model before thinking the multiplication operation as repeated addition. The responses of two pre-service teachers who used multiplication as area model and repeated addition are given in *Figure 2.* As indicated in *Figure 2*, the pre-service teacher in her third year of the program used the area model by firstly shading 2, two of 3 whole and one half of the last total, then shading the two fifth of each whole. In addition, it was also seen that the pre-service teachers preferred to use the rectangular area as the area model on their answer sheets. On the other hand, another pre-service teacher in his first year of the program used the repeated addition for multiplication of fractions stated on the answer sheet as "add $\frac{2}{5}$ twice and then add one half of $\frac{2}{5}$ with itself together".

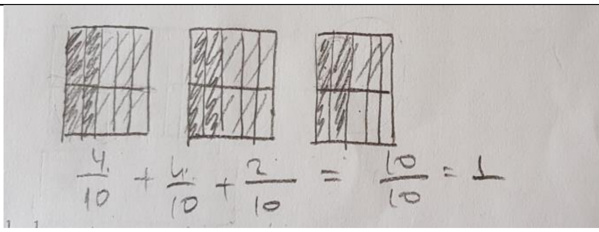
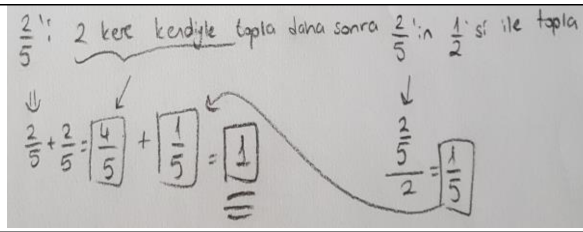
Area model	*Repeated addition*
	

Figure 2. *Examples of strategies for multiplication of fraction*

Differently from the strategies given above, approximately 10% of the pre-service teachers got the solution using the distributive property of multiplication over addition. They firstly expressed the mixed fraction as whole number and proper fraction then used the distributive property of multiplication over addition. An answer from a pre-service teacher, in her third year, is shown as follows:

$$"\frac{2}{5}\times 2\frac{1}{2} = \frac{2}{5}\times\left(2+\frac{1}{2}\right) = \left(\frac{2}{5}\times 2\right)+\left(\frac{2}{5}\times\frac{1}{2}\right) = \frac{4}{5}+\frac{2}{10} = \frac{4}{5}+\frac{1}{5} = 1".$$

When the pre-service teachers' strategies based on their grade level were considered, it was found that repeated addition and area model were the most commonly used strategies by pre-service teachers in their third year. As it is seen in *Table 3*, around 68% of the pre-service teachers who used the repeated addition strategy were generally the pre-service teachers in their third year. 88,8% of the pre-service teachers who used area model were again the pre-service teachers in their third year.

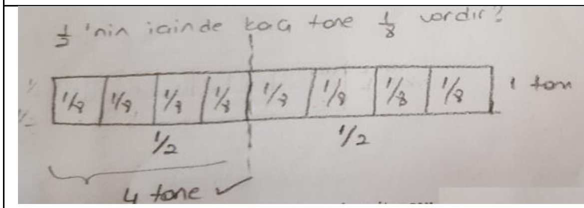
Table 3
The percentages of strategies used by pre-service teachers in multiplication of fractions based on their grade level

	Strategies for Multiplication of Fractions f (%)					
Grade Level	Area Model	Set Model	Repeated Addition	Distributive	Standard Algorithm	Incorrect or no Answer
1^{st} grade	1 (5,5%)	0 (0%)	1 (3,2%)	0 (0%)	8 (61,5%)	31 (33,7%)
2^{nd} grade	1 (5,5%)	2 (100%)	9 (29%)	11 (64,7%)	5 (38,5%)	39 (42,4%)
3^{rd} grade	16 (88,8%)	0 (0%)	21 (67,7%)	6 (35,3%)	0 (0%)	22 (24%)
Total	18 (100%)	2 (100%)	31 (100%)	17 (100%)	13 (100%)	92 (100%)

On the other hand, the strategies used for division of fractions, as it is seen in *Table 2*, only 27,6 % of the pre-service teachers gave incorrect or no responses when comparing to the percentages of incorrect or no response for multiplication. When the strategies were examined, the most commonly used strategy was the strategy of area model.

Specifically, 44% of the pre-service teachers preferred to use the area model. As the analysis of answer sheets revealed, the majority of pre-service teachers who performed the division of fractions through the use of area model used the rectangular or square area in their solution as also used in the multiplication of fractions. Accordingly, they primarily drew a shape and then specified one half of the whole in the rectangular area. Later, they divided the shape into 8 equal parts and tried to find how many $\frac{1}{8}$ in $\frac{1}{2}$. The examples of answers in which the area model was used for division of fractions are given in *Figure 3.*

Area model



Area model

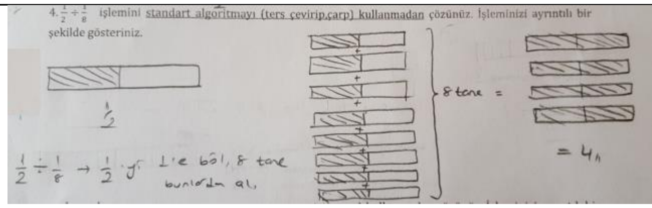
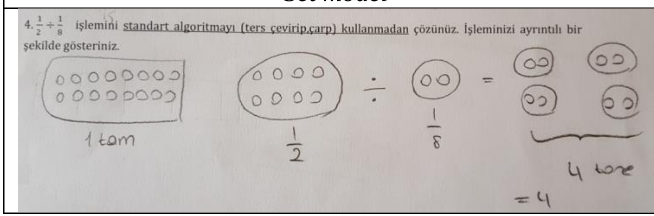


Figure 3. *Examples of strategies for division of fraction*

As indicated in the figure, both of the pre-service teachers used rectangular area model for the division of fractions. While one pre-service teacher in his third year looked for the fraction $\frac{1}{2}$ within $\frac{1}{8}$, another pre-service teacher in her second year used the area model by illustrating 8 times $\frac{1}{2}$ for the solution.

Beside the use of square and rectangular area, circle model was also used by pre-service teachers to illustrate the solution of division of fractions. Moreover, it was revealed that only 3,5% of the pre-service teachers used the set model. The responses of two pre-service teachers, one of whom used the set model for illustrating the solution, while the other of whom used the circle model in the area model, are presented in *Figure 4.*

Set model



Area model (circle model)

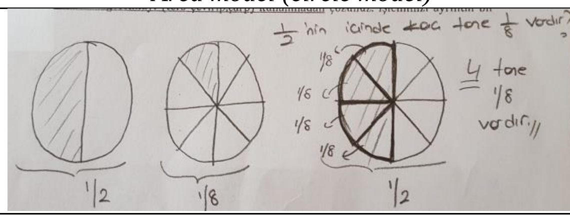


Figure 4. *Examples of strategies for division of fraction*

As seen in the figure, the pre-service teacher who solved the question through the set model drew 16 counters in a set expressing as a whole. After specifying the half of the whole with 8 counters and one eight of a whole with 2 counters, he drew 4 discrete sets, each consisting of 2 counters. In the second model in Figure 4, it is seem that the pre-service teacher concentrated on "how many groups of second fraction does the first fraction contain?" for the division of fractions.

After the use of model, the standard algorithm was the mostly preferred strategy by the pre-service teachers (14,5%). Even though the pre-service teachers were asked not to use the standard algorithm, they preferred to use it in their solutions. In this research, the standard algorithm is described by the rule 'invert second fraction and then multiply with first fraction' for division of fraction.

The strategies applied without using a model were common denominator, using decimal and repeated subtraction strategies. It can be stated that while 6,4% of the pre-service teachers used common denominator strategy for division of fractions, only almost 1% of them used repeated subtraction. In addition, 3,5% of them used decimal strategy. The pre-service teachers who used this strategy first converted the fractions into decimals and then divided each other. The response of a pre-service teacher in her second year is indicated as: "$\frac{1}{2}$= 0,5; and $\frac{1}{8}$= 0,125". Therefore, the answer is $\frac{1}{2}$/0,125 = 4". The responses of two pre-service teachers in their third year, one who used the common denominator strategy and the other who solved the division of fraction by using the division as repeated subtraction in whole numbers are given in Figure 5. The pre-service teacher who used the common denominator strategy wrote $\frac{1}{8}$ as $\frac{4}{8}$ and considered $\frac{4}{8}$ as 4 times $\frac{1}{8}$. Then, they looked for $\frac{1}{8}$ within $\frac{4}{8}$. On the other hand, the pre-service teachers who used the repeated subtraction strategy subtracted the $\frac{1}{8}$ four times from $\frac{4}{8}$.

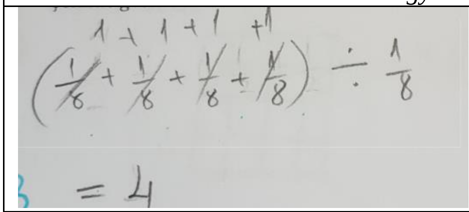
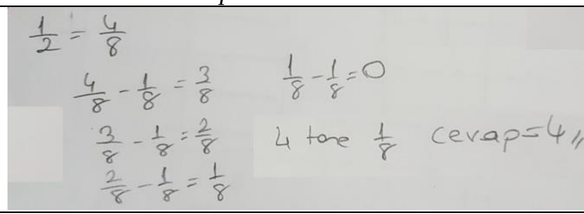
Common denominator strategy	*Repeated subtraction*
	

Figure 5. *Examples of strategies for division of fraction*

When the pre-service teachers' strategies based on their grade level are considered, the strategies used for division of fractions, the area model was generally one of the mostly preferred strategies that was used by the pre-service teachers in their third year (68,4%). Moreover, it was seen that approximately 64% of the pre-service teachers who used common denominator strategy was the pre-service teachers in their second year.

Table 4
The percentages of the strategies used by pre-service teachers in division of fractions based on their grade level

	Strategies for Division of Fractions f (%)						
Grade Level	Area Model	Set Model	Common Denominator	Repeated Subtraction	Using decimal strategy	Standard Algorithm	Incorrect or no Answer
1st grade	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	11 (44%)	25 (52,1%)
2nd grade	24 (31,6%)	3 (%50)	7 (63,6%)	0 (0%)	1 (16,7%)	13 (52%)	20 (41,7%)
3rd grade	52 (68,4%)	3 (%50)	4 (36,4%)	1 (100%)	5(83,3%)	1 (4%)	3 (6,3%)
Total	76 (%100)	6 (%100)	11 (%)	1 (100%)	6 (100%)	25 (100%)	48 (100%)

4. Conclusion and Discussion

The purpose of this study was to identify informal strategies used by pre-service elementary mathematics teachers for multiplication and division of fractions operation. The results showed that pre-service elementary mathematics teachers used four different strategies (using area model, using set model, repeated addition and distributive property) for multiplication of fractions operation and five different strategies (using area model, using set model, common denominator, repeated subtraction, using decimal strategy) for division of fractions operation except the use of standard algorithm. As mentioned before, these strategies used by pre-service teachers were described as informal and invented strategies in the literature (Son &Crespo, 2009; Van de Walle et al., 2007). In this study, using model was considered as an informal strategy since pre-service teachers tended to use model for reaching the solution as well as other identified informal strategies.

One of the significant findings of this study was that pre-service elementary mathematics teachers exhibited better performances in division of fractions than in multiplication of fractions when their performances were evaluated with regard to the category of incorrect or no response. In the current study, the pre-service teachers were asked to calculate the operation $\frac{1}{2} \div \frac{1}{8}$. Their high performances in division of fractions can be influenced by the nature of the question. This finding is similar to the results of some studies conducted in this field in the literature. In a study carried out by Işıksal and Çakıroğlu (2008), it was found out that the pre-service elementary mathematics teachers can divide the fractions easier when the dividend is bigger than the divisor; however, they expressed that they cannot make sense of the operation when the dividend is smaller than the divisor. In other words, it is easy to find the small fraction within the bigger fraction. This result supports the suggestion of Schwartz (2008) that the difficulty level of questions should be assessed with regard types of fractions as mixed, proper or improper fraction. Another reason might be that even though the pre-service teachers were asked not to use standard algorithm in division of fractions, some of them solved the questions using the standard algorithm. Use of standard algorithm might reduce the percentage of incorrect response.

Another important result of this research was that the most commonly used strategy after incorrect or no response was repeated addition in multiplication of fractions, and the area model in division of fractions. Moreover these strategies were generally preferred by pre-service teachers on their third year. It was an expected result that these strategies were mostly preferred by pre-service teachers on their third year. The first researcher was the responsible for the courses of Methods of Teaching Mathematics I and II, which pre-service elementary mathematics teachers attend in their 3^{rd}year during two semesters. These courses are compulsory that the pre-service teachers attend in their undergraduate program. In these courses, the first researcher created a learning environment which would help the pre-service teachers discover their knowledge and skills on the basic subjects of mathematics (four operations, fractions, operations in fractions, decimal representation, proportional reasoning) by organizing various activities. In these classes, different strategies in multiplication and division of fractions were emphasized and many discussions were held with the pre-service teachers. On the other hand, courses related to mathematics education that the pre-service teachers attend in their first and second year of undergraduate are limited. Therefore, it was expected that pre-service teachers were more familiar with these strategies when compared to other grade levels, which also complies with the findings of Işıksal and Çakıroğlu (2008) and Toluk-Uçar (2009).

Nevertheless, the percentages of using of strategies were quite low. It was evident that these strategies were not adequately conceptualized, since they were rarely used by the pre-service teachers even in their third year. Although they were asked not to use the standard algorithm, some pre-service teachers used the standards algorithm in multiplication and division of fractions. The pre-service teachers should primarily understand the concepts of operations of fractions in order to complete the conceptual learning process in the operations of fractions.

Another important result in the study was that approximately half of the pre-service teachers used the area model in division of fractions. This finding is parallel to the findings of other studies in the literature. Toluk-Uçar (2009) found out that pre-service teachers mostly prefer to use the area model as a graphical representation in operation of fractions. Moreover, she determined that the pre-service teachers were not able to use different other representation models. Ma (1999) stated that the pre-service teachers cannot represent the division of fractions using the models or made incorrect representations, and claimed that this is caused because the pre-service teachers could not perceive different solution methods of division of fractions. The findings of this study also revealed that few pre-service teachers preferred to use the set model except the area model. According to Tabak, Ahi, Bozdemir and Sarı (2010), representation of fractions with the set model is more difficult than representation with area and linear models. This can be the reason why the pre-service teachers preferred to use area model instead of set model. Other reason might stem from the fact that pre-service teachers' interpretation of the fractions. As suggested in Toluk-Uçar's (2009) study, area model is more convenient for part-whole meaning of fractions. Therefore, the part-whole meaning of fractions might be a reason for pre-service teachers to have a tendency using area model to solve the question. More specifically, most of the pre-service teachers using area model might interpret the division of fraction as measurement model. When using measurement interpretation of division, pre-service teachers asked how many of the second fraction is in the first fraction:"How many 1/8s are there in ½?" (Son &Senk, 2010).The need for examining the reasons behind thesestrategies, representations and interpretations was confirmed in the literature. In order to address this need, interviews should be conducted with the pre-service teachers to investigate the reasons of their strategies for further research study.

This study has some limitations. One of them is related to the structure of multiplication and division of fractions question itself which might effect on preferences of pre-service teachers' strategies. Multiplication of fractions was limited by the questions of which the solution is a whole number and both multipliers were fractions. Division of fractions was limited by the questions of which both the divisor and dividend were fractions and the result is a whole number as well. In addition, these questions were not given to the pre-service teachers within a context (with a verbal problem). If they were asked within a context, the strategies and models used by pre-service teachers were expected to differ (Dixon & Tobias, 2013).

In conclusion, the findings revealed that it is highly important to design the teacher education programs in a way that will support these kinds of strategies and conceptual perceptions. Although the pre-service teachers discuss about these kinds of informal strategies during their undergraduate education program, they need to gain more experience and do more practices on the mathematical concepts.

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