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Students with Emotional and Behavior Disorders: A Review of Mathematics Interventions with a Focus on Subtraction

Davranışsal-Duygulanımsal Bozukluğu Olan Öğrenciler: Çıkarma İşlemi Üzerine Matematik Müdahaleleri İncelemesi

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Abstract
Students with emotional and behavioral disorders (E/BD) display severe social and academic deficits that can adversely affect their academic performance in mathematics and result in higher rates of failure throughout their schooling compared to other students with disabilities (Webber & Plotts, 2008). This is of great concern as students with E/BD often receive lower grades, fail more classes, and have higher drop-out rates and fewer employment opportunities (Bullock & Gable, 2006). This review looks at mathematics interventions with a focus on subtraction for students with EBD. Implications for future research are included.

Keywords: Mathematics, subtraction, emotional and behavioral disorders (E/BD), learning disabilities (LD), mathematics disabilities, mathematics strategies/interventions.

Öz
Davranışsal-duygulanımsal bozukluğu olan öğrenciler, matematikteki akademik performanslarını olumsuz etkileyen ve eğitim hayatları boyunca diğer engelli öğrencilere oranla daha yüksek başarısızlık oranları ile sonuçlanan ciddi sosyal ve akademik bozukluklar sergilemektedirler (Webber & Plotts, 2008). Bu durum çok önemlidir, çünkü davranışsal-duygulanımsal bozukluğu olan öğrenciler genellikle daha düşük not alır, daha fazla derste başarısız olurlar. Ayrıca, okulu terk etme oranları daha yüksektir ve daha az iş imkanlarına sahiptirler (Bullock & Gable, 2006). Bu çalışmada, davranışsal-duygulanımsal bozukluğu olan öğrenciler için çıkarma işlemi üzerine yapılan matematik müdahalelerini incelenmektedir. Çalışmada, gelecek çalışmalar için çıkarımlar da yer almaktadır.

Anahtar Kelimeler: Matematik, çıkarma işlemi, davranışsal-duygulanımsal bozukluk, öğrenme bozukluğu, matematik öğrenme bozukluğu, matematik strateji/müdahalesi.

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1. Introduction

Students with emotional and behavioral disorders (E/BD) have deficits in behavioral performance, academic achievement, and social skills that greatly interfere with their educational performance (Rutherford, Quinn, & Mathur, 2004). Due to the severe social and academic deficiencies that adversely affect their academic performance, students with E/BD demonstrate higher rates of academic failure that persist throughout their schooling as compared to other students with disabilities (Webber & Plotts, 2008). These students display chronic disruptive behaviors that are generally identified after repeated academic failure and/or chronic disruptive behavior (Kauffman, 2001). Their deficits tend to persist across grade levels and content areas (Nelson, Benner, Lane, & Smith, 2004). Compared to their peers in other disability categories, students with E/BD are more likely to have lower grades, fail more classes, be retained, be served in restrictive settings, and drop out of school (Bullock & Gable, 2006). They may also have fewer employment opportunities and increased involvement in the legal system(Cullinan & Sabornie, 2004).

An academic area of particular concern for students with E/BD is mathematics. Research shows that students with E/BD perform more than one year below their non-disabled peers(Cullinan, 2002) and achieve well below national averages in the U. S. in mathematics (Anderson, Kutash, & Duchnowski, 2001). Students with mathematical difficulties who perform poorly on computation skills such as subtraction are more at risk of having difficulty in life skills, such as the workplace and money, and maintaining a social life (McCloskey, 2007).The purpose of this literature review is to examine the variety of mathematical interventions that have been applied to teach subtraction to students with E/BD and mathematics difficulties (including those with learning disabilities, LD). All literature published in English in both U.S. and international journals dating back to the 1900's was reviewed, including classic studies. Literature was accumulated through progressive searches using EBSCO and Google Scholar using these search terms: mathematics, subtraction, subtraction with regrouping, emotional and behavioral disorders (E/BD), learning disabilities (LD), mathematics difficulties, mathematics disabilities, mathematics strategies/interventions, equal additions algorithm, borrow and payback, Australian method, and decomposition method. Ancestral articles were conducted from reference lists. All articles used in this literature review had been peer-reviewed with the exception of the articles from the early 1900's.

2. Defining E/BD

The Individuals with Disabilities Education Act (IDEA, 2004) in the U. S. has defined an emotional or behavioral disability as exhibiting one or more of the following characteristics: (a) inability to learn not explained by intellectual, sensory, or health factors; (b) inability to build or maintain satisfactory relationships with peers and teachers; (c) inappropriate behaviors or feelings under normal circumstances; (d) a pervasive mood of unhappiness or depression; and/or (e) a tendency to develop physical symptoms or fears associated with personal or school problems. These characteristics must have been displayed to a marked degree and over an extended period of time.

Behaviorally, students with E/BD may exhibit inappropriate classroom behaviors during academic tasks such as anxiety and nervousness (Liaupsin, Jolivette, & Scott, 2007), non-compliance (Osher et al., 2007), and/or verbal or physical aggression (Lane, 2007). Poor social skills such as difficulty in taking turns, appropriately seeking teacher attention, maintaining appropriate peer interactions, or responding appropriately in social situations (Cook et al., 2008)may affect their ability to complete academic tasks (Sutherland, Lewis-Palmer, Stichter, & Morgan, 2008). Students with E/BD face educational challenges in preparing and organizing materials, listening to lectures, taking notes, participating in class, mastering academic content, and studying for tests (Mastropieri & Scruggs, 2001).

3. General Instruction for Students with E/BD

Two common functions of problem behaviors are associated with poor academic performance for students with E/BD. Students may engage in inappropriate behaviors because the academic task is too difficult (skill deficit) and disruption allows them to escape the demand of the task (Colvin, 2004; Van Acker, 2002).Others may be able to successfully perform, but choose to engage in negative behaviors (performance deficit) (Witt, VanDerHeyden, & Gilbertson, 2007). While a skill deficit requires attention focused on instructional strategies, and a performance deficit requires attention directed at antecedents and consequences of behavior, incentives can be provided to increase productivity. Researchers have found that frequent feedback and praise from teachers and peers, as well as active responding and engagement, have positive effects on behavior and academic performance (Greenwood, 1996; Sutherland & Wehby, 2001).

4. Mathematics Performance of Students with Disabilities

It is estimated that 5 to 10% of students in primary schools have a mathematics disability (MD) and that nearly 50% of all students in the U. S. have difficulty in mathematics (Siegler, 2007). Students with MD demonstrate deficits in three areas: 1) procedural, 2) semantic memory, and 3) visuo-spatial. Furthermore, students who demonstrate difficulty in mathematics may or may not have been diagnosed with another disability (e.g., E/BD or mild intellectual disability). According to the *National Assessment of Educational Progress at Grades 4* (NAEP; 2015), a fourth-grader's mathematics assessment showed students with disabilities scored in the bottom of the *Basic* level of achievement compared to their typical peers. In the number properties and operations section of the mathematics assessment, they scored 20 points lower than their peers without disabilities.

It is well known that students with E/BD demonstrate deficits in mathematics achievement compared to their typical peers (McLaughlin, Krezmien, & Zablocki, 2009). This is not surprising as academic underachievement is one of the identifying criteria for E/BD (IDEA, 2004). Researchers have shown that students with E/BD demonstrate significant mathematics deficiencies in elementary school, performing 1 to 2 grade levels behind their peers (Templeton, Neel, & Blood, 2008). Greenbaum and colleagues (1986) found that 97% of students with E/BD, ages 12 to 14, were performing below grade level in mathematics. Consistent with these findings, Nelson et al. (2004) conducted a cross-sectional study of 155 K-12 students with E/BD and found they experienced academic achievement deficits in mathematics and that such deficits appeared to broaden over time.

5. Computational Skills

According to the U. S. National Council for Teachers of Mathematics (NCTM, 2015), computation skills are the basis for mathematical standards including number and operations, geometry, algebra, measurement, and data analysis and probability. A recent report from the National Mathematics Advisory Panel in the U. S. (NMAP; 2008) indicated that computational ability is dependent upon basic fact recall, which requires fluency with the standard algorithms for addition, subtraction, multiplication, and division. Furthermore, conceptual understanding of mathematical operations, fluent execution of procedures, and the ability to recall basic facts support effective and efficient problem solving.

Little is known about the underlying cognitive processes that contribute to math achievement and MD (Geary, 1993). A failure to develop number sense as well as numerical deficits and procedural errors has been found to be the primary cause of MD (Geary, 1993). Children with MD have 1) persistent deficits in some areas of arithmetic and counting knowledge, 2) immature understanding of certain counting procedures, 3) use problem solving strategies more commonly used by younger students, 4) commit more procedural errors, and/or 5) have difficulty retrieving basic facts from long term memory (LTM). These deficits can be understood as being related to a combination of disrupted functions of the central executive system, difficulties with information representation and manipulation in the language system, and/or result from compromised visuo-spatial systems. More recent research has focused on the role of working memory in mathematical cognition. Several researchers have found that working memory plays a crucial role in calculation and in solving arithmetic word problems such as subtraction (Alloway, Gathercole, Adams, Willis, 2005; Bull & Scerif, 2001; Geary, Hamson, & Hoard, 2000;Passolunghi & Pazzaglia, 2005). Students with MD have been found to lack the ability to automatically recall basic facts and identify errors when computing problems such as subtraction with regrouping (Geary, Hamson, & Hoard; Swanson & Jerman, 2006).

Failure to develop sound computational skills may impact students as they progress through school. As knowledge develops cumulatively in mathematics, the acquisition of basic skills is critical for students in the primary grades. According to Woodward (2004), as academically low-achieving students move through the early grades, they face a number of difficulties as they encounter increasingly complex mathematical tasks. A descriptive study by Calhoon, Emerson, Flores, and Houchins (2007) of high school students with mathematics disabilities (MD) reveals these students continued to show a lack of computational fluency in a majority of mathematics areas at the 4th grade level. More specifically, they demonstrated profound difficulties in subtracting multiple digits with regrouping. Results suggest that the lack of retention of fourth-grade-level computational skills presents difficulties in learning higher order math skills for students with MD.

6. Mathematics Interventions for Students with Disabilities

It is critical to identify effective instructional strategies in mathematics for students with E/BD (Pierce, Reid, & Epstein, 2004). Due to similarities in academic performance between students with LD and E/BD, with both groups demonstrating below-average performance in content areas, deficits in basic academics, and low motivation (Fulk, Bringham, & Lohman, 1998), researchers have suggested that instructional strategies effective for students with LD can also be useful for students with E/BD (Bauer, Keefe, & Shea, 2001).

Effective instructional interventions include positive interactions, high rates of engagement, self-monitoring, peer-assisted learning, organizational supports, and direct instruction (Spencer, Scruggs, & Mastropieri, 2003). Several meta-analyses have been conducted (Codding, Burns, & Lukito, 2011; Hodge, Riccomini, Buford, & Herbst, 2006; Kroesbergen & Van Luit, 2003) that compare various interventions across mathematics skills. Common findings suggest that while drill and practice (Goldman, Mertz, & Pellegrinio, 1986, 1989) and modeling (Fuchs et al., 2008) tend to be most effective, a combination of interventions leads to better outcomes (Codding et al., 2011).

Currently, there is a paucity of research on mathematics interventions aimed at improving academic performance of students with disabilities (Bryant et al., 2008).A recent meta-analysis of interventions of basic mathematics computation in single-case research was conducted by Methe, Kilgus, Neiman, and Riley-Tillman (2012).The analysis examined interventions for additions and subtraction; only 2 out 11 focused on subtraction. In a review of instructional interventions in mathematics for students with E/BD specifically, Hodge and colleagues (2006) identified 13 studies that addressed basic computation skills in mathematics. Interventions included self-regulation and self-management; peer tutoring; mnemonics; Concrete-Representation-Abstract (CRA); Cover, Copy, Compare (CCC); error analysis; direct instruction; and alternative algorithms, each of which will be discussed below. However, as stated earlier, only a few interventions have been proven to be effective in helping students with LD/EBD in subtraction and much more research is needed in this area.

6.1. Self-Regulation/Self-Management

Self-regulation is the ability to regulate one's cognitive activities (Flavell, 1976) and includes strategies such as self-instruction, self-questioning, self-monitoring, self-evaluation, and self-reinforcement (Montague, 2008). Dunlap and Dunlap (1989) evaluated the effectiveness of a self-monitoring package that was applied to two, three, and four digit subtraction with regrouping problems. Using a multiple baseline across students design, three students, ages 10 -13 with LD were provided with didactic explanations, verbal feedback, and a point incentive during the baseline condition and individualized self-monitoring checklists during the self-monitoring phase. During maintenance, the checklists were removed and students continued to work under the previous conditions. The results indicated the use of the self-monitoring package produced immediate and substantial gains for each student and held across the maintenance condition.

In 2000, Levendoski and Cartledge conducted a study with 4 boys, ages 9-11, with severe emotional disturbance who demonstrated substantially lower levels of on-task behaviors and academic productivity in mathematics to monitor their behavior while working independently on newly taught material. A single-subject design was used, consisting of baseline, self-monitoring, return to baseline, self-monitoring, and fading of self-monitoring. Students were taught in small groups in a self-contained class during their daily math period. Data were collected on on-task behavior completing math worksheets designed for students instructional levels during a 20-minute independent math period, which occurred following math instruction. Math worksheets contained 25-30 problems, ranging from addition and subtraction to long division. Self-monitoring cards were given to students during intervention phases. Results indicated that when self-monitoring was in place, all four students increased both on-task behavior and academic productivity.

6.2. Peer Tutoring

Peer tutoring strategies are designed to improve math performance and behavior through peer tutoring, group rewards, and self-management procedures. Students are assigned partners by the teacher and follow highly structured tutoring procedures. Tutors present material previously covered by the teacher and provide feedback to the tutee. Students take turns as the tutor and tutee while the teacher circulates the room. Examples of peer tutoring strategies include Peer Assisted Learning Strategies (PALS), a highly structured format designed by Fuchs and

Fuchs, 2001, and Reciprocal Peer Tutoring (RPT), designed to assist students working in small groups (Fantuzzo, King, & Heller, 1992).

Researchers have found peer tutoring to be effective in increasing academic performance in mathematics for children at different ability levels (Calhoon & Fuchs, 2003; Fuchs, Fuchs, Phillips, Hamlett,& Karns, 1995). Researchers have also investigated cross-age tutoring (Beirne-Smith, 1991), as well as within-class peer assisted learning (Calhoon & Fuchs, 2003), in computation for students with LD. In a meta-analysis, Gersten and colleagues (2009) found the studies above showed consistently more modest effect sizes than other mathematics interventions analyzed. However, results also indicated stronger findings for cross-age tutoring interventions. In 1995, Harper, Mallette, Maheady, Bentley, and Moore evaluated the effects of using class-wide peer tutoring with three elementary students with mild disabilities using an alternating treatment design to teach subtraction computational skills. Results indicated that peer tutoring was effective in increasing students' accuracy, rate of responding, and retention with basic subtraction facts.

6.3. Mnemonics

Mnemonics strategies typically refer to words, sentences, or rhymes to help students recall important information. They have been found to be effective for students with mathematics disabilities (Greene, 1999).Research suggests that mnemonic strategies may improve computational fluency as students can rely on remembering specific cues, rather than rehearsal and repetition (Maccini, Mulcahy, & Wilson, 2007).Examples of mathematics mnemonics include Slobs & Lamps and RENAME. Slobs & Lamps is designed to help students remember the regrouping process of borrowing and carrying. Slobs is used in subtraction where students follow a series of steps to solve a subtraction problem: 1) look at the top right number, 2) see if it is **s**maller or **l**arger than the lower number, 3) cross **o**ff the number in the next column, 4) **b**orrow one ten from that column by reducing the number by one and adding ten to the number in the right column, and 5) **s**ubtract the lower number from the top number. Another mnemonic used for solving subtraction problems is RENAME: a) read the problem, b) examine the one's column using the BBB phrase (i.e. Bigger number on Bottom, Break down ten and trade), c) note ones in the ones column, d) address the tens column, e) mark tens in the tens column, and f) examine and note hundreds; exit with a quick check (i.e. add product to the subtrahend to see if the total is equal to the minuend).

Manalo, Bunnell, and Stillman (2000) investigated the effects of using process mnemonics for teaching computational skills to 8th grade students with MD. In the first experiment, students were randomly assigned to process mnemonics, demonstration-imitation, study skills, or no instruction. In experiment 2, instructors were used to teach the skills. Students in the process mnemonics group made significant improvements in subtraction that were maintained through the follow up stage for both experiments.

6.4. Concrete-Representation-Abstract (CRA)

The CRA teaching sequence supports the learning of a variety of mathematical skills for students with LD using a graduated instructional sequence (Ketterlin-Geller, Chard, & Fien, 2008). The first stage consists of *concrete* or hands-on instruction using manipulatives. As students progress, pictorial *representations* of the previously used manipulative objects are introduced. The final *abstract* stage of instruction uses numbers and operational symbols to present mathematical concepts (Witzel, Riccomini, & Schneider, 2008).

Research has shown CRA to be effective for teaching basic mathematics facts, fractions, algebra, and place value to students with LD and E/BD, as well as students with MD(Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Flores, Hinton, & Strozier, 2014). In 2009, Flores studied the effects of CRA used to teach subtraction with regrouping to six third-grade students who were failing mathematics; four were identified as having LD. Using a multiple-probe across groups design, students received instruction 3 days a week for 30 minutes each day. Probes used to measure progress consisted of 30 two-digit minus two-digit subtraction with regrouping problems. Results indicated that CRA instruction produced academic gains in subtraction with regrouping across all students. Five of the six students maintained performance at or above the criterion level during maintenance.

Recently, Mancl, Miller, and Kennedy (2012) conducted a multiple-probe-across-participants study using the CRA sequence with integrated strategy instruction to teach subtraction with regrouping to 5 students in 4th and 5th grade with LD in mathematics. Instruction took place in a resource room for approximately 30 minutes a day. A total of 11 scripted lessons were used to guide the students in learning subtraction with regrouping. During the concrete lessons, 5 of the 11 lessons students used three-dimensional plastic base-ten blocks with 11x18 place value mats.

The next three lessons involved representational learning using drawings. One strategy lesson was incorporated teaching the students the steps to the mnemonic device RENAME. The final two lessons involved abstract level learning. All participants showed immediate gains during the intervention and achieved mastery of 80% or higher on all lessons.

6.5. Cover, Copy, Compare (CCC)

CCC is a self-managed strategy that has also been shown to be effective in mathematics (Skinner, Shapiro, Turco, Cole, & Brown, 1992). The five-step procedure provides students with increased opportunities to respond to mathematics material and self-evaluate their progress as follows:1) review a problem and its solution on the left side of the paper, 2) cover the problem and solution with an index card, 3) solve the problem on the right side of the paper, 4) uncover the problem and solution on the left side, and 5) evaluate their response and make corrections to the response if it is incorrect by copying the correct problem and response a number of times. CCC has been effective for increasing student engagement and providing immediate corrective feedback as it provides numerous opportunities to respond to academic stimuli (Skinner, Turco, Beatty, & Rasavage, 1989). Researchers have found using CCC increases fluency and accuracy in subtraction (Grafman & Cates, 2010; Mong & Mong, 2010).

In a meta-analysis conducted by Joseph and colleagues (2012), a total of 10 studies in mathematics with 180 participants with and without disabilities revealed that the CCC method was clearly effective in helping students improve their academic performance. This was especially true when this procedure was implemented with other research-based interventions such as goal setting or token economies.

6.6. Error Analysis

Error analysis has been used as an assessment strategy to identify specific errors in conjunction with other intervention strategies. Error analysis is the process of evaluating student's responses and identifying errors to help direct instruction, improve student outcomes, and make instructional and curriculum decisions (Mercer & Mercer, 1998; Riccomini, 2005). Research has shown that evaluating student work to identify errors is one of the main principles for remedial education for students with mild disabilities such as E/BD and LD (Salvia & Ysseldyke, 2004). Evaluating students' mathematical errors specifically can help improve student outcomes (Riccomini); provide valuable information for assessment, instruction, and curriculum development (Mercer & Mercer); provide modifications in instructional methodology; and provide information to develop a specific plan for teaching and learning (Fernandez & Garcia, 2008). Analysis of student performance increases understanding and the prediction of math performance (Balacheff, 1990). Analysis of students' mathematical errors allows teachers to focus on and correct only the cause of the specific difficulty instead of focusing on re-teaching the entire skill (Parmar & Cawley, 1997).

Since computation skills are essential for learning more complex mathematics, identifying errors in computation skills can be valuable in analyzing a student's procedural and computational knowledge and provide relevant information for instructional decisions (Resnick, 1984). Importantly, studies on calculation show most errors demonstrated by students are systematic (Graeber, 1992) and the result of mistaken or missing knowledge (Van Lehn, 1982). Students make errors more frequently as problems become more complex and involve multi-digit problems in computation (Calhoon et al., 2007). Errors in computation include fact errors, operation errors, procedural errors, wrong operation, defective algorithm, incomplete algorithm, grouping error, inappropriate inversion, identity error, zero error, random response, and careless error (Ashlock, 2006).

Although studies have shown that students with mathematics difficulties demonstrate difficulty with single- and multi-digit mathematical problems (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003), there is little research using error analysis to determine the type of errors they make (Raghubar et al., 2009). Only a few studies have focused on errors in subtraction among students with disabilities (Skrtic, Kvam, & Beals, 1983).

Among these, inversion errors have been found to be the most common type of all systematic errors. Inversion errors occur when the minuend is subtracted from the subtrahend in subtraction problems requiring regrouping, or borrowing (Buswell & John, 1926; Cox, 1975). In 1978, Blankenship investigated the acquisition, generalization, and maintenance of skills among 9 students with LD who made systematic inversion errors in subtraction when borrowing. Results indicated that using a demonstration plus feedback technique to teach the decomposition method of subtraction reduced students' inversion errors in subtraction. Overall systematic inversion errors decreased from 86.7% to 6.7% and accuracy increased from 0% to 86.2%. In 1982, Frank, Logan, and Martin investigated the subtraction errors of 94 elementary students with LD. In subtraction problems requiring regrouping, one of the most common errors was inversion. Sugai and Smith (1986) conducted an error analysis on the types of error made by seven students with LD in grades 3-5 using the equal additions algorithm to teach subtraction. Before training, 6 of the 7 students made the same type of error (reversing the order of subtraction) when computing subtraction with regrouping. After training, reversal errors decreased significantly.

6.7. Direct Instruction

Direct instruction is the explicit teaching of rules and strategies combined with immediate, corrective feedback through guided practice (Gersten, Carnine, & White, 1984). This approach is teacher led as the teacher controls the instructional goals and pace, chooses the appropriate materials, and provides immediate corrective feedback to the student. In a meta-analysis of mathematics interventions, Kroesbergen and Van Luit (2003), found direct instruction approaches to be effective for basic skills acquisition for students with disabilities, although only a few studies focused on the acquisition of mathematics skills. Researchers have investigated the use of an explicit instructional approach to teach multiplication preskills (Carnine, 1980), basic facts (Carnine & Stein, 1981), and word problems (Darch, Carnine, & Gersten, 1984). In 1986, Kameenui , Carnine, Darch, and Stein used a direct instruction approach to teach subtraction. Twenty-three first graders identified as low performers were randomly assigned to either direct instruction group or a comparison group. The strategy for the direct instruction group was a semi-concrete, line drawing strategy, using clearly articulated teaching sequences containing explicit, step-by-step teacher modeling and assessment of student mastery at each step. The comparison group was taught the concept of subtraction using pictures and teacher discussion. Results indicated that the students who received the explicit strategy benefited more than the students in the comparison group.

6.8. Alternate Algorithms

Alternative algorithms are strategies designed to improve academic performance. Several alternative algorithms/methods have been identified to help students compute subtraction problems more efficiently and effectively. These are the Austrian algorithm, counting-up algorithm, low-stress algorithm, the additive method, the inverse relation method, indirect addition, and the equal additions algorithm. In the U. S., three different algorithms were commonly used until the 1940s including the decomposition, the Austrian method, and the equal additions, each of will be discussed below as they are classic strategies.

6.8.1. Decomposition: The decomposition algorithm of subtraction is commonly known as the *borrowing* method. It requires a student to subtract the subtrahend from the minuend, borrowing from the tens and adding to the ones as needed to complete the problem (see Figure 1). While this decomposition, or take-away, method has been advocated for students with disabilities since the 1920's, it is inconsistent with the definitions of subtraction among many special educators.

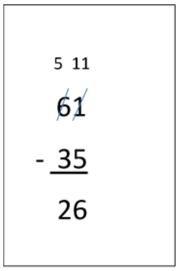

Figure 1 *Decomposition Method*

6.8.2. Austrian method: The Austrian method of subtraction is also known as the additions method. It makes a more precise connection between addition and subtraction in that it gets one to think of what needs to be added to the minuend to get the difference (Ross & Pratt-Cotter, 2000). In this method, the solution is found by directly relating the answer to addition. Students start with the smaller number and decide what number, when added to the smaller number, will give you the larger number. For example, when given the problem 13 – 7, the student should think, "7 and what gives you 13?" Finding the missing addend in this case helps connect the concepts of addition to subtraction.

Recent research has shown that indirect addition, which is similar to the Austrian method, is an efficient strategy for subtraction with small differences (Threllfall, 2002; Torbeyns, DeSmedt, Ghesquière, & Verschaffel, 2009). Using indirect addition, the solution to the problem is found by calculating the difference of two numbers. Students start with the smaller number and add, or count, up to the larger number. While indirect addition has been shown to be effective, it is rarely taught or used among traditionally schooled children (Torbeyns et al., 2009). In 2010, De Smedt, Torbeyns, Stassens, Ghesquire, and Verschaffel investigated the development of indirect addition as an alternative for solving multi-digit subtraction for 35 third-graders. Students were assigned to either an explicit or implicit learning environment that aimed to encourage the development of indirect addition. Results revealed that students in both groups rarely used the indirect addition method throughout the study. However, when indirect addition was used, it was executed very efficiently.

Furthermore, Selter (2001) conducted a study and found students used indirect addition on three-digit subtractions only 1% of the time. In a study where participants were assigned to either choice or no-choice groups, Torbeyns et al. (2009) found that in the no-choice condition, participants' who were instructed to apply the indirect addition strategy demonstrated significantly better performance in terms of speed and accuracy.

6.8.3. Equal additions algorithm: The equal additions algorithm can be traced back to the 15th and 16th centuries (Johnson, 1938) and is commonly referred to as *the borrow and repay* method. In this method of subtraction, a power of ten is borrowed to add to the necessary place in the minuend and repaid by adding to the digit in the next place of the subtrahend (see Figure 2). For example, when given the problem 95 – 28, 8 cannot be subtracted from 5, therefore 10 units must be added to 5 in the top number to form 15 and 10 units added to 20 in the bottom number which adds up to 30. The answer (67) remains the same because we added 10 to both the top number and bottom number. According to Ross and Pratt-Cotter (2000), this method is more representative of the term *borrow* than the decomposition algorithm, as a power of ten is *borrowed* from the minuend and then added to the subtrahend.

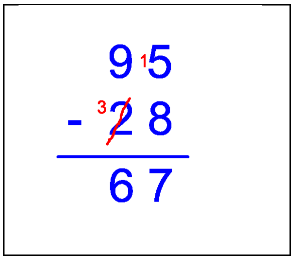

Figure 2 *Equal Additions Algorithm*

An extensive search of the literature has produced limited empirical research studies on subtraction with regrouping for students with E/BD or LD. From the limited research found, the equal additions algorithm has been shown to be as effective as, if not superior, to the decomposition algorithm in several studies (Brownell, 1947; Hoppe, 1975), and was the primary method of subtraction taught in the U. S. until the 1940's. Results from several classic studies (Ballard, 1914; McClelland, 1918; Winch, 1919) found that students in grades 2-5 with and without disabilities made significant gains using the equal additions algorithm over the decomposition algorithm.

More recently, Sugai and Smith (1986) conducted a study with seven students with mild disabilities in 3rd – 5th grades using the equal additions algorithm to teach subtraction with a specific modeling technique. Students received 15-minute instruction sessions daily on four types of subtraction with regrouping problems until mastery for three consecutive days at 90% was obtained. Those who made errors were corrected with oral prompting, referral back to the model, and the use of fingers. Results showed an increase in the percentage correct of subtraction problems requiring regrouping using the equal additions algorithm. To achieve 90% accuracy for any problem type, the minimum number of days required to teach the equal additions algorithm was three and the maximum was 13.

7. Summary

The limited research on academic interventions for students with E/BD reviewed by Nelson et al. (2004) found only 55 studies conducted in the past 30 years, most of which have focused on student-directed interventions rather than teacher directed ones (Hodge et al., 2006). As students with E/BD exhibit academic deficits, more research needs to be conducted to determine the effectiveness of current instructional programs and interventions (Nelson et al., 2004), particularly in mathematics. While researchers and educators are aware that many students experience difficulties in mathematics, instruction for these students has not received the attention given to reading instruction (Gersten et al., 2009).

Most research in the area of mathematics for students with E/BD has focused on basic mathematics skills, and has failed to investigate effective interventions in problem-solving and higher order mathematics skills. While most of the research on academic interventions in mathematics focuses on basic math fact recall, basic computational skills, and problem solving (Miller, Strawser, & Mercer, 1996; Montague, 2008), there is little research that addresses more advanced computational skills such as subtraction with regrouping.

When considering effective mathematics strategies for students with E/BD, an approach that incorporates multiple instructional techniques and strategies may be valuable (Jolivette et al., 2008; Verschaffel, Greer, & De Corte, 2007). For instance, teaching students with E/BD the equal additions method of subtraction using a direct instruction technique may be effective. In a meta-analysis of mathematics instruction for students with LD, Gersten and colleagues (2009) found that studies that incorporated direct, or explicit instructional strategies, resulted in significant effects and produced some of the largest effect sizes. Specifically, when studies focused on teaching a single mathematical proficiency or to solve a wide variety of problem types that included multi-digits, the results indicated large effects.

With regards to subtraction with regrouping, very little emphasis has been placed on defining effective instructional strategies to assist students who struggle with this concept. Of the three algorithms used to teach subtraction with regrouping, the decomposition method is clearly the most commonly used strategy in the U.S. However, many students continue to struggle with this concept. Alternate algorithms, such as the equal additions algorithm, may prove to be an effective alternative for students with disabilities.

8. Implications for Research

Of all students with disabilities, students with E/BD may present the most unique and challenging characteristics when it comes to improving academic outcomes. Due to the nature of the disability, these students often present behaviors that interfere with their academic success and consequently are often left to complete paper and pencil tasks in independent seat work. This seems to be especially true in more restrictive settings, where students' emotional and behavioral deficits are often a priority. As a result, much of the existing literature is aimed at providing self-regulation strategies that address behavioral and academic concerns. Academic strategies that improve student performance need to be further investigated. Additional research for students with E/BD using a systematic approach of direct instruction with equal additions and error analysis may be effective.

As educators search for effective mathematics strategies to support students who are struggling with higher-order math procedures such as subtraction with regrouping, it is important to choose strategies that have been proven to be effective. Strategies such as direct instruction have been proven over time and multiple disciplines to be effective in improving academic outcomes for students with LD and E/BD. Other strategies, such as the equal additions algorithm, may not have as much research to support effectiveness, but the research that exists, certainly shows benefit for students. Those who struggle to learn concepts with traditional strategies need alternative strategies to achieve academically. Combining multiple strategies, such as direct instruction and equal additions, to improve outcomes is an effective way to meet the needs of these struggling students and improve their educational outcomes.

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