BAŞKENT UNIVERSITY
JOURNAL OF EDUCATION

2020, 7(2), 250-266 ISSN 2148-3272

Pre-service Middle Grade Mathematics Teachers' Ability to Notice: The Case of Fractions

Ortaokul Matematik Öğretmeni Adaylarının Farkındalık Becerileri: Kesirler Durumu

Zülfiye Zeybek Şimşek[a]*

[a]*Tokat Gaziosmanpasa University, Tokat, Turkey*

Abstract
It is observed that recently, there has been an increase in the studies on the importance of including student ideas in classroom discussions.However, interpreting how students understand concepts and responding to them on the basis of their understanding in the moment is not an easy feat. The purpose of this study was to investigate pre-service mathematics teachers, grade 5-8, ability to notice student's understanding of the concept of fractions. In addition to investigating pre-service teachers' ability to notice, this study also aimed to analyze pre-service teachers' own understanding of fractions. A mathematics education course was designed to engage pre-service teachers with teacher noticing activities, which consisted of solving high cognitive level tasks, explaining student solutions for the same tasks and responding to the students on the basis of their understandings. The data included pre-service teachers' responses to a quiz that consisted of various open-ended fraction questions as well as classroom instruction videos during three weeks. The results of the study demonstrated that the pre-service mathematics teachers struggled with the concept of fractions, specifically writing story problems to illustrate fraction expressions or solving fraction expressions without using standard algorithms. Additionally, they also struggled with explaining students' solutions and deciding how to respond to the students on the basis of their understandings of fractions.

Keywords: Middle grade mathematics, pre-service teachers, student knowledge, teacher noticing

Öz
Öğrenci fikirlerinin sınıf içi tartışmalara dâhil edilmesinin önemine yönelik yapılan çalışmalarda son zamanlarda bir artış olduğu görülmektedir. Ancak, öğrencilerin kavramları nasıl anladıklarını yorumlamak ve şu anki anlayışlarına dayanarak onlara cevap vermek karmaşık bir beceridir. Bu çalışmanın amacı, ilköğretim matematik öğretmeni adaylarının öğrencilerin kesir kavramına yönelik bilgilerini fark edebilme becerisini araştırmaktır. Öğretmen adaylarının öğrenci düşünmelerini fark edebilme becerisinin yanında, bu çalışma ayrıca öğretmen adaylarının kesir kavramına yönelik kendi konu alan bilgilerini de analiz etmeyi amaçlamıştır. Bu amaç doğrultusunda, öğretmen adaylarına yönelik yüksek bilişsel düzeyli görevlerin çözülmesi, aynı görevler için öğrenci çözümlerinin açıklanması ve öğrencilere kendi anlayışlarına göre soru sorulması aşamalarının olduğu bir matematik dersi planlanmıştır. Çalışmanın veri kaynakları, öğretmen adaylarına yönelik planlanan ders videoları ve çeşitli açık uçlu kesir sorularından oluşan bir kısa sınavdan oluşmaktadır. Çalışmanın sonuçları, öğretmen adaylarının kesirler kavramına yönelik yaşadıkları zorlukları ortaya koymuştur. Öğretmen adaylarının özellikle kesir ifadelerini göstermek için hikâye problemleri yazma konusunda zorlandıkları görülmüştür. Ayrıca, çalışmanın bulguları öğretmen adaylarının kesir ifadelerini genellikle standart algoritmalar kullanarak çözdüklerini de gözler önüne sermiştir. Öğretmen adaylarının öğrenci çözümlerini açıklamakta ve kesirleri anlamalarına dayanarak öğrencilere nasıl cevap vereceklerine karar vermekte zorlandıkları da çalışmanın bulguları arasındadır.

Anahtar Kelimeler: Ortaokul matematiği, öğretmen adayları, öğrenci bilgisi, öğretmen farkındalığı.

**ADDRESS FOR CORRESPONDENCE: Zülfiye Zeybek Şimşek, Department of Mathematics, Faculty of Education, Tokat Gaziosmanpaşa University, Tokat, Turkey, e-mail: zulfiye.zeybek@gop.edu.tr, Tel: +90 (507) 194 33 28, ORCID ID: 0000-0003-1601-8654.* *Received Date: November 15th, 2019. Acceptance Date: February 18th, 2020.*

1. Introduction

Sherin (2001) states that "emphasis on understanding the ideas that students offer in class is one of the hallmarks of mathematics education reform" (p. 84). Many other researchers have also echoed the same theme (e.g. Jacobs et al., 2007; Jacobs, Lamb, & Philipp, 2010; Schifter, 2011; Sherin & van Es, 2009). Thus, incorporating student thinking in the ongoing instructions and organizing instructions around these ideas has gained an increased attention. Current reform efforts highlight the importance of making thoughtful instructional movement acts such as integrating students' ideas in the ongoing lesson (NCTM, 2000; NGA & CCSSO, 2010). The construct of noticing students' mathematical thinking and incorporating their thinking in instruction is, therefore, foundational to the complex view of teaching captured by these reform efforts (Jacobs, Lamb, & Philipp, 2010).

Teachers need to adapt to the ideas that students raise in class and therefore must listen to their ideas carefully, analyze them and respond to them in the moment. The development of these abilities of interpreting students' thinking may allow teachers to make appropriate instructional decisions (Chamberlin, 2005). However, interpreting how students understand mathematical concepts and responding to their understanding respectively, which is recognized as one of the essential components of teacher knowledge (Ball, Thames, & Phelps, 2008), is not an easy feat (Schifter, 2001; Steinberg, Empson, & Carpenter, 2004). Since teachers cannot preplan responses in the moment, this improvisational part of teaching requires teachers to constantly analyze and connect specific situations to what they know about students' mathematical development (Franke, Kazemi, & Battey, 2007; Lampert, 2001).

Although there is a growing body of research in the area of teacher noticing of student thinking, the majority of these studies tend to look quite broadly at the students' mathematical thinking in general (Walkoe, 2015). In order to leverage this construct, some studies have recently focused on characterizing the skill of noticing students' mathematical thinking in different domains such as pattern generalization (Callejo & Zapatera, 2016) or early numeracy (Schack, Fisher, Thomas, Eisenhardt, Tassel, & Yoder, 2013). Thus, this study is embedded in this line of research and focuses on addressing pre-service mathematics teachers' (PSMTs') ability of noticing a specific mathematical concept —fractions, a well-established concept to constitute a struggle for students at all levels (Behr, Lesh, Post & Silver, 1983). Sherin (2002) claims that if teachers' own understanding is insufficient, they will be unable to interpret and respond to students' understandings productively. Thus, in addition to investigating PSMTs' ability of noticing, this study also aims to investigate what PSMTs' conceptions of fractions are.

Specifically, this study addresses the following two research questions:

1- What are the pre-service mathematics teachers' conceptions of fractions?

2- How do the pre-service mathematics teachers explain students' strategies and respond based on their understanding in the case of fractions?

1.1. What is Teacher Noticing?

There are many events that take place at once in a classroom and teachers need to be able to identify these key moments during instructions. Doing so requires that teachers should be flexible in their instructions and notice moments of student thinking that can be used to advance instruction. Research has shown that the strategies that students use can provide a window into their understandings (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003; Lester, 2007). Thus, teacher noticing of student understanding is and should be a key component of classroom instructions. One of the professional skills that pre-service mathematics teachers should develop as a component of becoming a mathematics teacher is the ability of noticing student understanding (Mason, 2002; Sherin et al., 2010). Tirosh (2000) argues that a major goal in teacher education programs should be to promote pre-service teachers' knowledge and ability of understanding students' common thinking ways about the mathematics topics that the teachers will teach.

Teacher noticing has been an essential topic of interest recently in research on teacher practice (Mason, 1998; Santagata et al., 2007; Sherin et al., 2011; Star & Strickland, 2007; van Es & Sherin, 2010). Mason (2002) considers teacher noticing to be a fundamental element of expertise in teaching, characterized by: (a) keeping and using a record, (b) developing sensitivities, (c) recognizing choices, (d) preparing to notice at the right moment, and (e) validating with others. Van Es and Sherin (2002) also consider noticing consisting of several abilities including (a) identifying noteworthy aspects of a classroom situation, (b) using knowledge about the context to reason about classroom interactions, and (c) making connections between specific classroom events and broader principles of teaching and learning.

Jacobs et al. (2010) conceptualize teacher noticing as a set of three interrelated skills: (1) attending to students' strategies, which refers to the extent to which teachers attend to the mathematical details in students' strategies; (2) interpreting students' mathematical understanding- the extent to which the teachers' reasoning is consistent with both the details of the specific students' strategies and the research on students' mathematical development; and (3) deciding how to respond on the basis of students' understanding which refers to the extent to which teachers use what they have learned about the students' understanding from the specific situation and whether their reasoning is consistent with research into students' mathematical development. Tirosh (2000) focuses on student understanding of mathematical concepts by differentiating two terms— (1) knowing-that and (2) knowing-why. She describes knowing-that as research-based knowledge about students' common conceptions (as well as misconceptions) and ways of thinking about mathematical concepts. Knowing-why, on the other hand, refers to general knowledge about possible sources of these conceptions and to the understanding of the sources of student's responses, which constitutes indeed an essential aspect of interpreting student understanding. These interrelated skills of teacher noticing served as a lens to investigate pre-service teachers' ability to explain student strategies as well as to respond on the basis of their understanding of fractions (see Table 2 for how these skills were integrated in the framework used in this study).

1.2. Why Fractions?

Fractions are well known to constitute a struggle for students (Behr, Lesh, Post & Silver, 1983). One of the difficulties for such struggle comes from the multifaceted notion of fractions, which includes part whole, quotients, measures, ratio, rate, and operators (Behr et al., 1983; Brousseau, Brousseau, & Warfield, 2004; Kieren, 1993). For instance, the fraction 3/4 can be seen as a part of a whole (three out of four equal parts), as a quotient (three divided by four), as an operator (three quarters of a quantity), a ratio (three parts to four parts), and as a measure (as a point on a number line). As a result, fraction concepts are often considered the least understood topic in school.

Prior research state that students struggle with fractions as a result of generalizing strategies learned and used for whole numbers, encountering interference from knowledge of rote procedures, or building upon prior informal knowledge regarding to fractions (e.g. Behr, Lesh, Post, & Silver, 1983; Clarke & Roche, 2009; Mack, 1990; Pearn & Stephens, 2004). Additionally, students' informal knowledge of fractions is often disconnected to their knowledge of fraction symbols, procedures or representation. Mack (1990) argues that students come with prior informal knowledge on operations (addition, subtraction, division, or multiplication), which could not be drowned upon when it comes to fractions. Similarly, Tirosh (2000) state that intuitively based mistakes such as the divisor must be less than the dividend could serve as constraints for students when it comes to fractions.

Studies concluded that similar results in fraction knowledge continue to exist for pre-service and in-service teachers (e.g. Ma, 1999; Tirosh, 2000). Researchers evidenced pre-service teachers' limited knowledge of fractions on division (e.g. Ball, 1990; Tirosh, 2000; Simon, 1993) as well as addition, subtraction, multiplication of fractions (e.g. Newton, 2008). In addition to having difficulties with the concept of fractions, studies also concluded that teachers have difficulty in explaining students' thinking and reasoning with fractions (e.g. Tirosh, 2000). All these results encouraged to consider the concept of fractions as the focus area for this paper.

2. Method

2.1. The Mathematics Education Course

In order to investigate pre-service mathematics teachers' (PSMTs)' ability to notice students' mathematical thinking while analyzing their own content knowledge, a mathematics education course was designed. The course was a three-credit university course and took 14 weeks of instructions during the spring semester of 2016. During the semester, the PSMTs were engaged in several activities in three strand areas—Geometry, Algebra, and Number Sense. The PSMTs were engaged in class activities in three steps including solving high level tasks —procedures with connections and doing mathematics as described by (Smith & Stein, 1998)—, sharing and discussing different solutions, and analyzing students' thinking through presented student artifacts, excerpts and student videos. During the last step of analyzing student work for the same tasks, the PSMTs were asked to (1) explain students' solutions and their understandings of the concepts in details and then (2) respond to the student(s) by listing at least two questions that should be asked next as guided by Jacobs et al. (2010).

Figure 1. Classroom Instruction Steps

2.2. Participants

A class of 58 pre-service middle grade teachers, who are going to be certified to teach grades 5 through 8 mathematics, enrolled in a three-credit mathematics education course in the spring semester of 2016. The PSMTs were sophomores when this study took place. The participants were selected by using purposeful sampling. Purposeful sampling can be defined as choosing the sample from easily accessible and practicable units due to the limitations in time, money and labor (Patton, 1990).

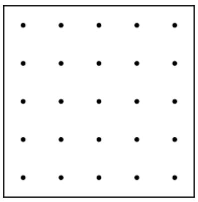
2.3. Data Sources

The purpose of the study was to investigate the PSMTs' conceptions of fractions and their ability to notice student understandings of fractions. The PSMTs were engaged in fraction activities for three weeks during the semester. The activities used during instructions aimed at conceptualizing fractions and fraction operations. Sample of the tasks used during instructions were displayed in Table 1 below. The data for this study mainly came from these three weeks. The instructions, during three-hours of instruction time, were videotaped by a stable camera, which was located at the back of the classroom to capture the board. Additionally, an unannounced quiz with open-ended fraction questions, which were adapted from existing literature, was administered to the PSMTs in order to investigate their subject matter knowledge of fractions as well as their ability of noticing student understandings of fractions. The quiz was administered at the beginning of the fraction unit instructions. The PSMTs were given an hour to complete the questions in the quiz. Given that the purpose of the study was to document the PSMTs' conceptions of fractions as well as their ability of noticing student thinking, not to capture the effectiveness of a designed course, the responses of the PSMTs to the quiz questions and their responses during instructions were analyzed with no tendency of documenting the progress of the PSMTs' reasoning skills.

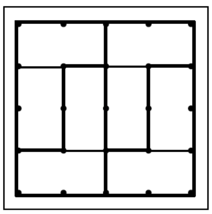
Table 1
Sample fraction activities used during instructions

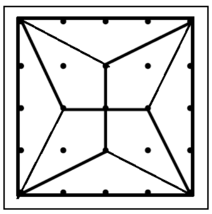
TASK A:
How many ways can you show eighths on your Geoboard?

- Use as many rubber bands as you need to divide your Geoboard into1/8ths.
- Do this in as many different ways as you can.
- Record each of your solutions on geodot paper.
- Compare your results with the rest of your group's.
- Be ready to explain why each of your solutions shows eighths.

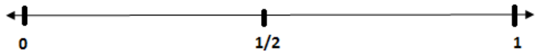


Sample Student Work:
Compare the two responses presented below. Do you think they both are correct or not? Why? Why not?





TASK B:
Put the fractions at appropriate places on the number line below. Please justify your responses.



$^{5}/_{9}$, $^{3}/_{8}$, $^{5}/_{11}$, $^{5}/_{6}$, $^{5}/_{12}$, $^{7}/_{8}$, $^{11}/_{12}$, $^{4}/_{7}$

Sample Student Work:
Okan argued that 5/6 and 7/8 are the same since they both have one piece left to be a whole. How would you respond to Okan? Why?

TASK C:
Solve each question below. Then, draw a model to illustrate each operation.

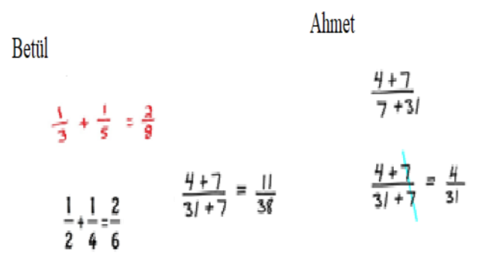
a- $^{1}/_{3} + {}^{1}/_{5}$ =?
b- $^{1}/_{2} + {}^{1}/_{4}$ =?
c- $^{4}/_{7} + {}^{7}/_{31}$ =?

Sample student work:
Explain the students' responses presented below. How would you respond to those students?

TASK D:
Solve each question below. Then, write a story problem that could be used to illustrate each operation.

a- $^{1}/_{2} \times {}^{3}/_{4}$ = ?
b- $^{2}/_{3} \times {}^{5}/_{6}$ =?
c- $^{2}/_{4} \times {}^{3}/_{9}$ =?
d- $175 \times {}^{2}/_{5}$ =?
e- $^{4}/_{6} \div {}^{3}/_{2}$ =?
f- $^{1}/_{2} \div 4$ =?
g- $^{3}/_{5} \div {}^{1}/_{5}$ =?
h- $^{6}/_{7} \div {}^{1}/_{14}$ =?

Sample Student Work:
Ayşe solved $^{6}/_{7} \div {}^{1}/_{14}$ as $\frac{6 \div 1}{14 \div 7} = \frac{6}{2} = 3$. What do you think about her strategy? How would you respond to Ayşe?



Six questions in the quiz were included to investigate the PSMTs' conceptions of fractions as well as their ability to notice student thinking (see Appendix A for quiz questions). Among the questions that investigated the PSMTs' conceptions of fractions, one question asked the PSMTs to solve the given fraction division expressions, one question asked the PSMTs to write a story problem that could be solved by the given fraction multiplication expression and the remaining questions asked the PSMTs to evaluate whether the provided models and story problems could be used to illustrate the given fraction expressions. The questions, that investigated the PSMTs' ability to notice student thinking, aimed at investigating the interrelated skills of teacher noticing: (1) explaining students' strategies and (2) deciding how to respond on the basis of students' understanding. The PSMTs' responses to the quiz questions as well as the instruction videos during the three weeks served as the data sources for this study.

2.4. Data Analysis Process

Given that the purpose of this study was to investigate the PSMTs' knowledge of fractions and their ability of noticing student understanding of fractions, the study was designed as a qualitative study. Strauss and Corbin (1990) define qualitative studies, as any kind of research that produces findings not arrived at by means of statistical procedures or other means of quantification. Content analysis approach guided the analysis of data in this study. Krippendorff (2013) defines content analysis as a research technique for making replicable and valid inferences from texts to the contexts of their use (p. 24). Flick (2014) explains that one of the essential features of content analysis is the use of categories, which are often derived from theoretical models—categories are brought to the empirical material and not necessarily developed from it—although they are repeatedly assessed against it and modified if necessary. In this study, the categories are adopted from the interrelated skills that were proposed by Jacobs et al. (2010). The data analysis process occurred in three phases. First, the videos of the instructions during the three weeks were viewed by the author. After the first review of the videos, the parts where the PSMTs were engaged in fraction activities and teacher noticing skills of describing and deciding how to respond were selected and transcribed. In the second phase of the data analysis process, the selected segments and the transcripts were viewed again by the author and the PSMTs' responses in the components of teacher noticing- explaining and responding- were coded in four hierarchical levels as following: Level 0: No response/Non-relevant, Level 1: Lack of Evidence, Level 2: Limited Evidence, and Level 3: Robust Evidence (see Table 2). These four levels were also applied to code the PSMTs' responses to the quiz questions that were asked to assess the PSMTs' ability to notice student understanding. Then, the transcripts of the selected segments and the PSMTs' responses to the quiz questions were coded by a second coder. The second coder was familiar with the literature related to fractions and teacher noticing. The second coder and the author then compared and contrasted their coding results. All disagreements were solved through discussion.

Table 2
Levels of teacher noticing

Levels of Thinking	Explaining Students' Strategies	Responding on the Bases of Students' Understandings
Level 0: No response/Non-relevant	At this level, the PSMTs left out the question blank or their responses were not relevant with what the question was asking.	At this level, the PSMTs left out the question blank or their responses were not relevant with what the question was asking.
Level 1: Lack of Evidence	At this level, the PSMTs only provide superficial descriptions of students' strategies with no supporting evidence from student work.	At this level, the PSMTs do not base their responses on students' understandings. Instead, they chose to ask broad questions that could be implemented in any situations.
Level 2: Limited Evidence	At this level, the PSMTs provide some details of students' strategies, but left out some important information or details.	At this level, the PSMTs base their responses on students' understandings. However, their responses were based off of little evidence regarding to student understanding.
Level 3: Robust Evidence	At this level, the PSMTs provide detailed descriptions of students' strategies and supported their descriptions by providing evidence from student work.	At this level, the PSMTs base their responses on students' understandings. The PSMTs provided robust evidence regarding to student understanding.

Finally, in the third phase of the data analysis process, the PSMTs' responses to the quiz questions, which were used to assess the PSMTs' conceptions of fractions, were viewed and coded as correct versus incorrect and standard algorithm versus alternative method by the author first. Later, the second coder coded the responses individually. The author and the second coder came together to compare and contrast the individual codes. This process again continued till all the individual codes were aligned entirely. "Reverse and multiply" strategy for fraction division, "multiply across" strategy for fraction multiplication and "common denominator" strategy for fraction comparison problems were accepted as standard algorithms since those strategies were mostly used in school mathematics (van de Walle, Karp & Bay-Williams, 2013).

3. Findings

In this section, the results related to the first research question—subject matter knowledge of the pre-service teachers in the concepts of fraction—will be presented first. Then, the results related to the second research question—the pre-service teachers' ability to notice student understanding of fractions— will be presented next.

3.1. Pre-service Mathematics Teachers' Subject Matter Knowledge of Fractions

The responses of the PSMTs' to the quiz questions and the percentages for each category were presented cumulatively in the Table 3 below.

Table 3
Percentages of pre-service teaches responses to fraction questions

Codes	Quiz Q3	Quiz Q 5	Quiz Q 5-a	Quiz Q 6-a	Quiz Q 6-b	Quiz Q 6-c
#of the PSMTs (Code)	%92 (Standard Algorithm)	%51 (Incorrect)	%67 (Incorrect)	%31 (Incorrect)	%87 (Incorrect)	%18 (Incorrect)
#of the PSMTs (Code)	%8 (Alternative Method)	%49 (Correct)	%33 (Correct)	%69 (Correct)	%13 (Correct)	%82 (Correct)

As can be seen in the table, the PSMTs struggled with writing and/or evaluating story problems to illustrate fraction multiplication/division expressions the most. For instance, in quiz question 5-a, the PSMTs were asked to write a story problem to illustrate a fraction multiplication expression: $1\frac{1}{2}x\frac{2}{3}$. Only 33% of the PSMTs were able to write a problem that could be used to illustrate the given fraction multiplication expression while 67% of the PSMTs failed to construct a problem that could be solved with the given expression. Consistent with previous studies (e.g., Ma 1999), these findings indicate that pre-service teachers had much more difficulty constructing story problems to illustrate fractions expressions than they had solving those expressions.

The PSMT in Figure 2 wrote a problem as follows: "Ayse has decided to bake cookies based on a recipe, which requires $1\frac{1}{2}$lt warm milk for cooking a tray of cookies. If she bakes $\frac{2}{3}$ of a tray of cookies, how much milk will she need?" The PSMT's problem was coded as a correct response since it was asking $\frac{2}{3}$ of $1\frac{1}{2}$ and could be solved with the given expression.

Ayşe decided to make a cookie by looking at a cookie recipe. One tray of cookies requires $1\frac{1}{2}$ lt of warm milk. How much milk will you need to make $^2/_3$ tray cookies?†

Figure 2. A Correct Response to the Quiz Question 3

The PSMT's response in Figure 3, on the other hand, was coded as an incorrect response. The problem wrote by the PSMT in Figure 3 was coded as an incorrect response since the answer of the problem was indeed provided in the problem by stating that the sizes of each piece would be $\frac{3}{2}$; therefore, there was no problem to be solved by the given expression. Further, the PSMT started the problem with dividing $1\frac{1}{2}$ pie as if she was constructing a division problem rather than a multiplication problem.

If we divide $1\frac{1}{2}$ pie into $\frac{3}{2}$ slices, what is the size of each slice=?

Figure 3. An Incorrect Response to the Quiz Question 3

Another incorrect response to the quiz question 5-a was presented in Figure 4 below. In this response, the PSMT attempted to use division as the inverse operation of multiplication. It was apparent that the PSMT was attempting to use partitive interpretation of division, which caused the PSMT to write a story problem that did not make much sense in real life. Given that the partitive interpretation of division is more common for conceptualizing division, the PSMT attempted to construct a division problem by using this interpretation. However, when it comes to fraction division, the measurement interpretation is the one which is more helpful rather than partitive interpretation.

How do we distribute $\frac{3}{2}$ cake evenly among $\frac{3}{2}$ people?

Figure 4. Another Incorrect Response to the Quiz Question 3

In addition to struggling with writing a problem to illustrate fraction expressions, the results also demonstrated that the PSMTs struggled with representing fraction expressions by using a correct model. In quiz question 5, the

† The responses of the PSMTs were written in Turkish and translated to English by the author of the study. The translations were checked by another person who was fluent in English to make sure that the translations were coherent with the original responses.

PSMTs were asked to select an incorrect model to illustrate the given fraction multiplication expression. As can be seen in the Table 3, 51% of PSMTs failed to decide which models could be used to demonstrate the given expression. Among these incorrect responses, the majority of the PSMTs selected the model presented in D (see Appendix A)—the number line model—as an incorrect model. Given that the most commonly used model in textbooks is area model to represent fractions, selecting the length model to represent a fraction expression, as an incorrect model is not surprising.

The results of the quiz questions also demonstrated that the majority (92%) of the PSMTs used a standard algorithm—invert and multiply method— to solve the fraction division expressions in the quiz. The tendency of using standard algorithms to solve the fraction questions was also apparent during the classroom instructions.

The PSMTs struggled to solve the fraction questions when they were specifically asked not to use standard algorithms. In the classroom episode below, the PSMTs were engaged in comparing fractions. Not surprisingly the first method they attempted to use to compare different fractions was usually the common denominator method. However, when they were asked to use different methods, they struggled to come up with different strategies.

Instructor: How can you compare $\frac{5}{6}$ and $\frac{7}{8}$?
PSMTs: We can use the common denominator.
Instructor: Yes, we can use the common denominator method. What about any other methods? Without using the common denominator, how can you compare these two fractions?
Ömer: We can also use fraction strips
Instructor: Yes, we can do that. What else?
Yılmaz: $\frac{7}{8}$ is bigger because there are 6 glasses of a drink and you drank 5 glasses. There 8 glasses and you drank 7 of them. Of course, 7 is more.

Yılmaz attempted to compare the numerators of both fractions— $\frac{5}{6}$ and $\frac{7}{8}$ —without attending the size of each piece. In other words, he detached the numerators from the fractions and only focused on the numerators instead of treating the fractions as single entities. Orhan, on the other hand, was able to use a correct strategy to compare the fractions.

Orhan: Let's say we have two cakes at the same size and partition one into 8 pieces and the other into 6 pieces. Then we get 5 pieces out of 6 and 7 pieces out of 8. We can compare the leftovers. If the left over is smaller then the fraction will be bigger.

Instructor: So you say that $\frac{1}{8}$ is smaller than $\frac{1}{6}$, how do you know?

PSMTs: The bigger the number of partitions, the smaller the pieces. When you partition the same whole into more pieces, the pieces will get smaller.

The PSMTs' struggle with the concept of fractions was apparent in their responses to the quiz questions as well as during classroom instructions. They also struggled to describe and respond to students, which will be addressed next.

3.2. Pre-service Mathematics Teachers' Ability of Noticing Students' Understandings of Fractions

The PSMTs' responses to these questions, which were used to investigate their ability to notice students' understanding of fractions, and the percentages for each level, were presented cumulatively in the Table 4 below.

Table 4
Percentages of Pre-service Teachers to Quiz Questions of Teacher Noticing

	Quiz Q1		Quiz Q2		Quiz Q3		Quiz Q4
	Explaining	Responding	Explaining	Responding	Explaining	Responding	Explaining
Level 0	%36	%21	%21	%10	-	%5	%10
Level 1	%64	%40	%15	%59	%3	%79	%54
Level 2	-	%18	%51	%10	%97	%16	%31
Level 3	-	%21	%13	%21	-	-	%5

The results that were gathered from coding the PSMTs' responses to the quiz questions showed that the majority of the PSMTs' responses cumulated at Level 1 or Level 2. Furthermore, the percentages of the responses that were coded at level 2 were higher in the explaining questions than the percentages of the responses at Level 2 in the responding questions. Even though some of the PSMTs' responses were coded at level 3 in either explaining or responding questions, they were significantly lower than the responses that were coded at Level 1 and Level 2. The PSMTs' responses in each level for both explaining and responding type questions will be demonstrated next.

In quiz question 4, the PSMTs were asked to evaluate a student's strategy to calculate fraction division. Even though the strategy was mathematically sound, some of the PSMTs, who were coded at level 0, claimed that the strategy was incorrect. Below is a sample response of the PSMTs who claimed that the strategy was incorrect (see Figure 5). The PSMT provided $\frac{\frac{1}{2}}{2}$ as a counterexample—incorrect way of applying Raşit's strategy to a new situation. However, the strategy could be applied to all situations even the one that the PSMT provided. The PSMTs who thought that the strategy was wrong argued that at some point Raşit would get stuck because there would be a fraction in the denominator or numerator. Similarly, the majority of the participants who claimed that the method was wrong argued that this method was only ''applicable to the problems in which both the numerator and denominator of the dividend are divisible by those of the divisor.''

The rule of Raşit is, of course, wrong. Looking at his calculations, an error occurs on the way he goes.

$\frac{\frac{1}{2}}{2} = \frac{1}{2} \div 2 = \frac{\frac{1}{2}}{2} = \frac{\frac{1}{2}}{2}$ it continues like this and cannot be concluded. The fraction goes on.

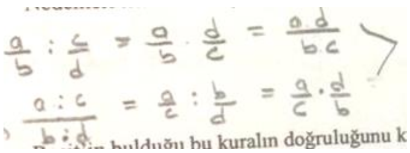
*Figure 5.*A Sample Response of the Quiz Question 4 Coded as Level 0

Some of the PSMTs were able to evaluate the presented student strategy in quiz question 4 as a correct strategy. However, they could not proceed to go beyond stating whether the strategy worked as it was evidenced in Figure 6. The PSMT in Figure 6 only stated that the strategy was correct but did not provide further explanation regarding to why it was a correct strategy. Thus, the PSMT's response was coded as Level 1.

Yes, it is true.

Figure 6. A Sample Response of the Quiz Question 4 Coded as Level 1

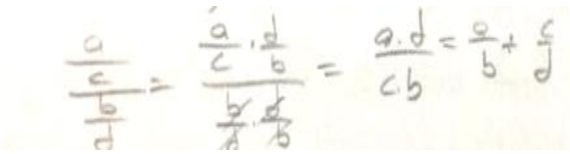
The PSMT, in the following response in Figure 7, attempted to check the validity of the strategy. However, the PSMT's way of checking validity of the student's strategy did not go beyond by simply applying the standard algorithm. Therefore, her response was coded as Level 2.



The formula is correct because the results are the same and it is seen that it is true for all fractions.

Figure 7. A Sample Response of the Quiz Question 4 Coded as Level 2

The PSMT's response in Figure 8, on the other hand, went beyond describing the steps of the algorithm, which is not an easy feat (Ball, 1990). The PSMT used the definition of multiplication of fractions and several mathematical principals (i.e. the product of a number and its reciprocal is 1). This method was referred as complex fraction by Tirosh (2000).



The rule is correct. In multiplication and division, precedency does not matter. Thus, the result will be true.

Figure 8. A Sample Response of the Quiz Question 4 Coded as Level 3

Tirosh (2000) explains the inverse operation as presenting the division as the inverse operation of multiplication. For instance, if $\frac{a}{b} \div \frac{c}{d}$ =x, then, x.$\frac{d}{c}$= $\frac{a}{b}$. Similarly, the PSMT in his response in Figure 9 used the inverse strategy to justify that the student strategy was correct.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} . \frac{d}{c}$ Yes, it is true because in order to change the places of d and c, we need to use the inverse operations. So, $\frac{a}{b} . \frac{d}{c} = \frac{a}{b} \times \frac{\frac{1}{c}}{\frac{1}{d}} = \frac{\frac{a}{c}}{\frac{b}{d}}$

Figure 9. Another Sample Response of the Quiz Question 4 Coded as Level 3

The PSMTs' responses to the quiz questions also demonstrated that they struggled with responding to the students based on their understanding of the concept. For instance, in the quiz question 1 when the PSMTs were asked to respond to Tanya by stating questions, the majority of them provided questions based on no evidence of understanding of Tanya's thinking. While many PSMTs chose to leave the question blank, some PSMTs provided responses that were indeed not aligned with Tanya's strategy. For instance, in her response in Figure 10, the PSMT chose to ask Tanya to compare the fractions of $^1/_8$ and $^1/_5$. Then, the PSMT claimed that Tanya would answer as $^1/_8$ is bigger than $^1/_5$ since 8 is bigger than 5. However, in the question scenario Tanya stated that if the difference between the denominator and the numerator of a fraction increases, then the fraction gets smaller. Thus, according to Tanya's strategy, she would indeed respond to the question as $^1/_5$ is bigger than $^1/_8$ since the difference between the denominator and the numerator of $^1/_8$ is bigger. There was no mention of comparing only the denominators of given fractions as claimed by the PSMT in Figure 10. Therefore, the PSMT's response in Figure 10 was coded as a non-relevant response since it was based off incorrect analysis of the presented student strategy.

I would ask Tanya to compare $^1/_8$ and $^1/_5$. Tanya would probably say that $^1/_8$ is bigger than $^1/_5$ because 8 is bigger than 5. Then I would ask her whether I would eat more cake when I divide a cake into 8 pieces or 5 pieces?

Figure 10. A Sample Response of the Quiz Question 1 Coded as Level 0

General probing questions such as "can you explain your response?" or "why did you use this method?" were common among the PSMTs. However, these types of questions require limited or no improvisational analysis of students' strategies. Instead, they could be asked in any circumstances. Therefore, these types of questions were coded as Level 1 since they included no evidence of students' thinking.

1) How do you arrive at this conclusion?
2) What do you think about the correctness of the answer?

Figure 11. A Sample Response of the Quiz Question 1 Coded as Level 1

The PSMT's response in Figure 12, on the other hand, was based off of student's understanding of fractions. The PSMT was able to address the cases where Tanya's strategy would not hold true and stated the limitations of the strategy by asking questions. In her first question, the PSMT asked Tanya to compare $^5/_7$ with $^{11}/_{14}$ in which Tanya's strategy would not hold true. Although the difference between the denominator and the numerator in the fraction of $^5/_7$ is smaller than the difference between the denominator and the numerator of $^{11}/_{14}$, $^5/_7$ is smaller than $^{11}/_{14}$. In her second question, the PSMT chose two equivalent fractions. Although the fractions were the same, according to Tanya's strategy, given that the difference between denominator and numerator in $^{4}/_{8}$ is bigger, Tanya would probably answer that $^{3}/_{6}$ is bigger. Later, the PSMT asked Tanya to compare four fractions in which the difference between denominator and numerator of each fraction is the same. Thus, the PSMT were able to analyze Tanya's strategy correctly and stated three questions in which her strategy would not hold true. Furthermore, the response of the PSMT in Figure 12 demonstrated not only evidence of understanding student thinking but the response in Figure 12 also demonstrated an understanding of whether the student would apply this incorrect strategy to other questions where this strategy would be wrong. Thus, this response was coded as Level 3.

→ How would you compare $^{5}/_{7}$ and $^{11}/_{14}$?

→ How would you compare $^{4}/_{8}$ and $^{3}/_{6}$?

→ How would you compare the fractions of $^{1}/_{2}$, $^{3}/_{4}$, $^{7}/_{8}$ and $^{10}/_{11}$?

Figure 12. A Sample Response of the Quiz Question 1 Coded as Level 3

4. Conclusion and Discussion

This study investigated pre-service mathematics teachers' subject matter knowledge of fractions as well as their ability of noticing student understanding in the concept of fractions. Across the data, it was apparent that the PSMTs struggled with the concept of fractions especially when they were engaged in high-level tasks such as constructing story problems or using models to illustrate fractions expressions. Furthermore, it was evident in the results of this study that the PSMTs struggled with explaining student solutions and responding to the students on the basis of their understanding.

There is an increased attention that calls to engage students in mathematical discussions whereby they share, discuss, and critique each other's ideas (NCTM, 2000; NGA & CCSSO, 2010). Doing so requires teachers to not only have profound understanding of the concepts that they teach, but also an ability to adapt to the ideas that students raise in class. However, studies that focus on teachers' knowledge reported a concern regarding elementary teachers' knowledge of mathematics (Ball, 1990; Ma, 1999; Simon, 1993), especially regarding to the concept of fractions (Ma, 1999). The results of this study demonstrated that although the PSMTs were able to perform well on solving questions in which they could use standard algorithms, it was not the case when they were engaged in high-level tasks such as writing story problems to illustrate fraction expressions. For instance, the results showed that only 33% of the PSMTs were able to write a problem that could be used to illustrate the given fraction multiplication expression while 67% of the PSMTs failed to construct a problem that could be solved with the given expression. Similarly, the PSMTs also struggled to model the given fraction expressions. The results demonstrated that 51% of PSMTs failed to decide which models could be used to demonstrate the given fraction multiplication expression. Smith and Stein (1998) state that solving high cognitive level tasks (procedures with connection and doing mathematics) are important vehicles for building students' capacity for deeper understanding of the concepts and these tasks require some degree of thinking, which means they cannot be solved mindlessly (p. 348).

Increased attention to student thinking has been shown to promote opportunities for student learning (Schifter 1998; Carpenter et al. 1989; Jacobs et al. 2007; Franke & Kazemi, 2001). Having knowledge of students' common conceptions as well as misconceptions about specific subject matter is essential for teaching (NCTM, 1989, 1991). Existing studies, however, have reported that pre-service teachers' abilities to analyze the reasoning behind students' responses were poor (Ball, 1990; Even & Markovitz, 1995; Even & Tirosh, 1995). The result of this study aligned well with the result of those studies by documenting that pre-service mathematics teachers' struggle with describing, analyzing student responses as well as responding to them based on their understandings. The results that were gathered from coding the PSMTs' responses to the quiz questions demonstrated that the majority of the PSMTs' responses to the question that required explaining the presented student work were coded at level 2 while the majority of their responses to the question that required responding to the students were coded at level 1.

This study has implications for teacher educators as well as for researchers. First, this study along with other studies (e.g., Ma, 1999; Tirosh, 2000) highlights the importance for teacher educators to consider how to help pre-service teachers learn to address students' mathematical thinking and to respond to them based on their understanding of the concepts effectively. If students' thinking is to be more commonly emphasized by current education reform movements, it is necessary to study teachers' ability of noticing student reasoning and to use the information gleaned to inform teacher education programs. However, as evidenced in the results of this study, it is not an easy feat. Thus, this study suggests that one of the major goals of teacher education programs should be to promote pre-service teachers' knowledge and ability of understanding students' thinking about mathematical concepts as suggested by Tirosh (2000). One possible way to achieve this ideal could be to provide various opportunities to pre-service teachers to engage in tasks that not only require them to solve high cognitive level tasks, but also to analyze student reasoning to the same tasks as employed by the study. Additionally, the framework used in this study for identifying pre-service teachers' explaining and responding levels (see Table 2) could be an essential tool for teacher educators as they plan and implement similar types of tasks in their instructions. This study also suggests that mathematics education research should continue to investigate pre-service as well as in-service teachers' ability to analyze students' thinking. And to respond to students based off of their thinking.

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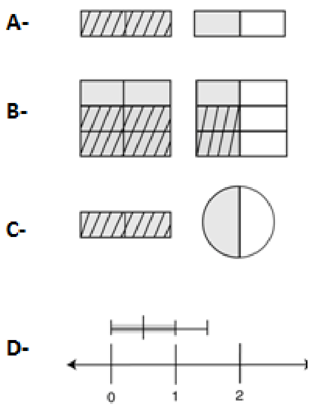
Appendix A:

Question 1: While investigating the subject of comparing fractions in the classroom, the teacher has given the students enough time to think about the following problem: $^1/_6$ $^2/_3$ $^5/_8$ $^1/_5$ $^2/_4$ Tanya argues that $^2/_3$ is the biggest one and then, $^2/_4$, $^5/_8$, $^1/_5$ and $^1/_6$. Because, if the difference between the denominator and the numerator increases, then the fraction gets smaller. a-Describe Tanya's strategy to compare fractions. b-If you were Tanya's teacher, what question(s) would you ask to Tanya? (adapted from Zaskis & Chernoff, 2008)
Question 2: Ali's response to the question above is as follows: Ali: 1/5 is bigger than 1/6. Because if you partition something up into 5 pieces, each piece will be bigger than if you partition it into 6 pieces. Then, If you compare 5/8, 2/3 and 2/4, 2/3 will be the biggest. Because, it was divided into thirds and the others were divided into fourths and eighths. Additionally, there is one piece left to be a whole in 2/3, there are two pieces left in 2/4 and 3 pieces left to be whole in 5/8. Thus, the fractions from biggest to smallest will be as follows: 2/3, 2/4, 5/8, 1/5, 1/6 a- Describe Ali's strategy to compare fractions. Do you think his strategy is correct? Why? Why Not? b- If you were his teacher, what question(s) would you ask to Ali?
Question 3: Answer the questions below. Please explain how you found your answers. (1) ¼ ÷4=…………………………….. (2) 1/4÷3/5=………………………… (3) 320÷1/3=………………………..… a- What kind of mistakes would you expect that a seventh grader might make while solving these questions? b- Why do you think seventh graders might make those mistakes? c- How do you think teachers should respond to those students who made those mistakes? (adapted from Tirosh, 2000)
Question 4: Raşit claims that he has found a rule to calculate fraction divisions. His rule is as follows: $\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$ Raşit argues that his rule is correct because, $\frac{1}{2} \div \frac{1}{2} = \frac{1 \div 1}{2 \div 2} = 1$ or $\frac{1}{8} \div \frac{2}{4} = \frac{1 \div 2}{8 \div 4} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$. a-Describe Raşit's strategy to divide fractions. Do you think it is true? Why? Why not?

(adapted from Tirosh, 2000)

Question 5: Which model below cannot be used to illustrate $1\frac{1}{2} \times \frac{2}{3}$?
(Mark One Answer)



(MKT Released Items, 2008)

a- Write a story problem that could be used to illustrate : $1\frac{1}{2}x\frac{2}{3}$

Question 6: Which of the following questions could be used to illustrate $1\frac{1}{4} \div \frac{1}{2}$? (Mark Yes, No, I'm Not Sure for each possibility.)

A- You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?

Yes **No** **I'm Not Sure**

Reason:

B- You have 1.25 TL and may soon double your money. How much money would you end up with?

Yes **No** **I'm Not Sure**

Reason:

C- You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick =1/2 cup) will you need?

Yes **No** **I'm Not Sure**

Reason:

(MKT Released Items, 2008)